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HULLBORNE HYDROFOIL SIX-DEGREE OF FREEDOM

MOTION PREDICTION AND COMPUTER PROGRAM - ZERO SPEED CASE

by

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and

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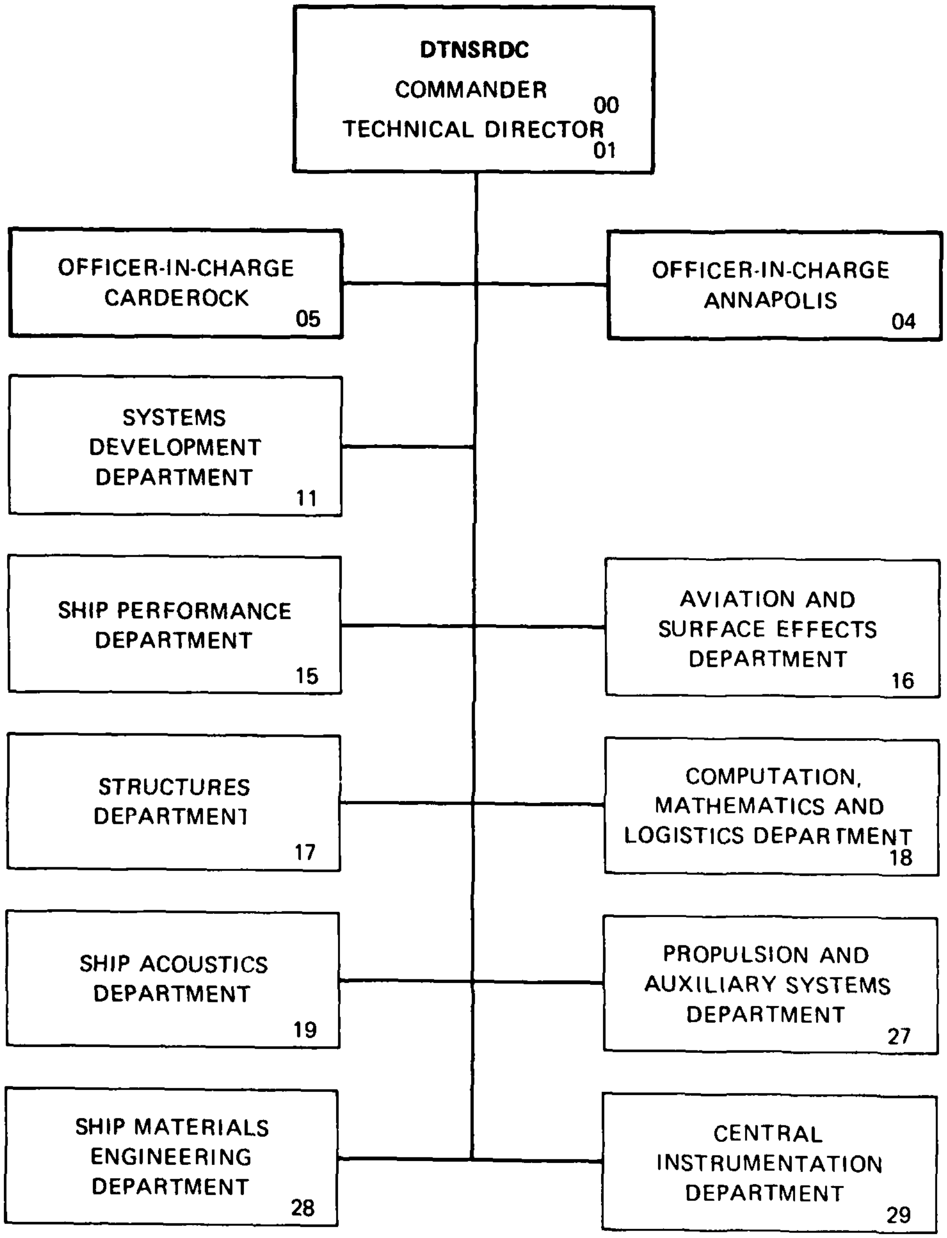

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Modifications were made to a motion prediction computer program for a hullborne hydrofoil in waves to include the zero speed case. The program can now be used to compute the six-degrees-of-freedom (6DOF) hullborne hydrofoil craft motions for a craft speed of zero as well as for a constant forward speed, less than the critical "lift-off" speed, with arbitrary heading in regular waves. Basically, the program consists of the already existing "DTNSRDC 6DOF Ship-Motions and Sea-Loads Computer Program" ~~1~~, 2, which (AD-A0[illegible])

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computes the motions of hulls in the displacement mode, with appropriate modifications to incorporate the effects of the submerged foil system on craft motions. Presented in this report are a discussion of the mathematical model for the zero craft speed case, input information pertaining to the hydrofoil system, and a discussion of the results. The Appendices present a listing of the program update cards and the subroutines pertaining to the foil system.

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LIST OF FIGURES (CONT)

NOMENCLATURE

A_j	inertial related force on a foil element in the j'th degree of freedom
A_{jk}	added mass coefficients
AR	aspect ratio
B_j	viscous damping related force on a foil element in the j'th degree of freedom
B_{jk}	damping coefficients
C_j	wave-making damping force on a foil element in the j'th degree of freedom
C_D	viscous damping coefficient for a foil element
C_w	wave-making damping coeffieient for a surface piercing foil element
F_j	wave excitation
I_{jk}	inertia coefficients
V_N	normal velocity at the foil element's midpoint due to craft motion
$\hat{V}_N$	complex amplitude of V_N
$\hat{W}_N$	complex amplitude of the normal wave velocity component at the foil element's midpoint
X, Y, Z	location of foil element's midpoint in the body system
c	chord
i	$\sqrt{-1}$
$\bar{i},\bar{j},\bar{k}$	unit vectors in x,y,z body system
k	wave number ω^2/g
m	mass
m_{jk}	generalized mass
$\bar{n}$	unit vector normal to foil element's midpoint
$\bar{r}$	position vector in body frame

NOMENCLATURE (CONT)

s	foil span
t	time
v	velocity in body reference frame
v_0	velocity in inertial system
v_N	normal velocity component on the foil element due to craft motion
w_N	normal wave velocity component on the foil element
x_0, y_0, z_0	coordinates of inertial reference frame
x, y, z	body coordinate system with z passing through CG and z=0 in waterplane
x', y', z'	foil element coordinate system with origin at midpoint and element in z'=0
Γ	foil dihedral angle
α	foil angle-of-attack
ε_j	phase lag in j'th mode
ζ	wave amplitude
η_j	motion displacement
$\hat{\eta}_j$	complex amplitude of $\eta_j(t)$
λ	wavelength
μ	heading angle of incident waves
ρ	fluid density
ϕ	wave velocity potential
ω	orbital wave frequency

ABSTRACT

Modifications were made to a motion prediction computer program for a hullborne hydrofoil in waves to include the zero speed case. The program can now be used to compute the six-degrees-of-freedom (6DOF) hullborne hydrofoil craft motions for a craft speed of zero as well as for a constant forward speed, less than the critical "lift-off" speed, with arbitrary heading in regular waves. Basically, the program consists of the already existing "DTNSRDC (David W. Taylor Naval Ship Research and Development Center) 6DOF Ship-Motion and Sea-Load Computer Program"[1*], which computes the motions of hulls in the displacement mode, with appropriate modifications to incorporate the effects of the submerged foil system on craft motions. Presented in this report are a discussion of the mathematical model for the zero craft speed case, input information pertaining to the hydrofoil system, and a discussion of the results. The Appendices present a listing of the program update cords and the subroutines pertaining to the foil system.

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INTRODUCTION

A description of the "DTNSRDC Hullborne Hydrofoil 6DOF Motion Prediction Computer Program" for the zero craft speed case is presented in this report. It serves as an extension to the previously developed program in Reference 2, which predicts the motion for a hullborne hydrofoil craft in six-degrees-of-freedom (6DOF) advancing at a constant forward speed in the displacement mode with submerged foils at an

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*References listed on Page 31.

arbitrary heading in regular waves. The program is an adaption of the already existing "DTNSRDC 6DOF Ship-Motion and Sea-Load Computer Program"[1], based on the theory by Salvesen, Tuck, and Faltinsen[3], which was developed for the prediction of motions and dynamic loads of conventional displacement type hulls, and is utilized for planing hulls in the displacement mode as well. The program modifications consist of adding the hydrodynamic force terms attributed to the immersed hydrofoil system to the already existing equations of motion for the hull. This technique has been used successfully by R.T.Schmitke who developed two computer programs for simulating the motions of a hullborne hydrofoil vessel at zero speed as well as for forward speeds less than the critical "lift-off" speed for head seas[4] and beam seas[5].

The hull related input information for the herein presented program remains identical to the original, "DTNSRDC 6DOF Ship-Motion and Sea-Load Computer Program." The additional input information relating the hydrofoil system is given for computing the hullborne hydrofoil motions either at zero speed or at a forward speed less than the critical "lift-off" speed for an arbitrary heading.

The modified program's output consists of the amplitudes and phases in surge, sway, heave, roll, pitch and yaw for a given set of wave frequencies and a specified set of forward ship speeds and headings, as well as the corresponding response amplitude operators, nondimensional transfer functions, and nondimensionalized excitation forces and moments as in the original program for ship motions. Also printed are the coefficients of motion and the excitation forces for the hydrofoil system as well as for the hull. Optionally, one can obtain the two sets of coupled differential

equations of motion in matrix form; one for surge, heave, and pitch and the other for sway, roll, and yaw. Each set is given for the hull portion, the foil-strut portion, the combination of the two, and the final inverted matrix with the solutions. Both sets are for the minimum specified frequency.

MATHEMATICAL MODEL

The analytical model for determining the motions of a hullborne hydrofoil craft at zero speed is obtained by adding linear and quasi-linear terms to the hull terms obtained by strip theory which is based on the solution of the sectional two-dimensional problem utilizing a close-fit source-distribution technique developed by W. Frank[1,6]. The major assumptions and limitations are:

1) wave amplitude of unidirectional regular waves and craft displacements from equilibrium are both small.

2) both the craft's beam and draft are much smaller than its length

3) the craft, both hull and hydrofoil system, are laterally symmetrical

4) dynamic lift attributable to the hull's planing surfaces is insignificant

5) the foil system is divisible and representable by a set of rectangular foil elements

6) interaction between the hull and hydrofoil system is negligible as is the interaction between the foil elements

7) the hydrofoil system's contribution to surging is negligible.

HULL EQUATIONS OF MOTION

The conventions used in the hullborne hydrofoil craft motion program

are the same as the "DTNSRDC Ship-Motion and Sea-Load Computer Program." The following will briefly restate the definitions used[3]. As shown in Figure 1, the vessel-oriented, right-handed coordinate system is defined to have its origin in the plane of the undisturbed free water surface. The positive x axis passes through the craft's stern, the y axis is to starboard, and the z axis is vertically upward passing through the center of gravity. The heading angle μ is defined to be 0 degrees for waves approaching the stern and 180 degrees for waves approaching the bow as illustrated in Figure 2. The encounter frequency, ω_e, being equal to the circular wave frequency ω for zer craft speed is given by the expression

$$\omega = \frac{\sqrt{2\pi g}}{\lambda} \tag{1}$$

for depths much greater than the wavelength λ.

With the assumption that the equations of motion are either linear or have been linearized, motion displacements can be expressed as

$$\eta_j = a_j \cos(\omega t - \varepsilon_j);\ j = 1,\ldots,6 \tag{2}$$

where a_j are the amplitudes and ε_j are the phase lags of the motion with respect to the maximum wave elevation above the origin. The subscripts j = 1...6, refer respectively to the translatory displacements of surge, sway, and heave and the angular displacements of roll, pitch, and yaw. Following from the above assumptions, the six linear coupled differential equations of motion can be written in complex form as:

$$\sum_{k=1}^{6} (m_{jk} + A_{jk})\ddot{\eta}_k + B_{jk}\dot{\eta}_k + C_{jk}\eta_k = F_j e^{-i\omega_e t};\ j = 1,\ldots,6 \tag{3}$$

where m_{jk} are the components of the craft's generalized mass matrix, A_{jk} are the added-mass coefficients, B_{jk} and C_{jk} are the complex damping and

restoring coefficients and F_j are the complex amplitudes of the excitation forces and moments.

For a hullborne hydrofoil craft as well as for a conventional displacement hull the six coupled equations of motions can be separated into two sets of equations. With the exclusion of hydrostatic restoring coefficients that are equal to zero for both the hull and foil system, the first set of three coupled equations of motion in surge, heave, and pitch is

$$\text{Surge}\quad (A_{11} + m)\ddot{\eta}_1 + B_{11}\dot{\eta}_1 + A_{13}\ddot{\eta}_3 + B_{13}\dot{\eta}_3 + A_{15}\ddot{\eta}_5 + B_{15}\dot{\eta}_5 = F_1 e^{-i\omega_e t} \tag{4}$$

$$\text{Heave}\quad A_{31}\ddot{\eta}_1 + B_{31}\dot{\eta}_1 + (A_{33} + m)\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 + A_{35}\ddot{\eta}_5 + B_{35}\dot{\eta}_5 + C_{35}\eta_5 = F_3 e^{-i\omega_e t} \tag{5}$$

$$\text{Pitch}\quad A_{51}\ddot{\eta}_1 + B_{51}\dot{\eta}_1 + A_{53}\ddot{\eta}_3 + B_{53}\dot{\eta}_3 + C_{53}\eta_3 + (A_{55} + I_5)\ddot{\eta}_5 + B_{55}\dot{\eta}_5 + C_{55}\eta_5 = F_5 e^{-i\omega_e t} \tag{6}$$

and the second set of equations is

$$\text{Sway}\quad (A_{22} + m)\ddot{\eta}_2 + B_{22}\dot{\eta}_2 + (A_{24} - mZ_G)\ddot{\eta}_4 + B_{24}\dot{\eta}_4 + C_{24}\eta_4 + A_{26}\ddot{\eta}_6 + B_{26}\dot{\eta}_6 + C_{26}\eta_6 = F_2 e^{-i\omega_e t} \tag{7}$$

$$\text{Roll}\quad (A_{42} - mZ_G)\ddot{\eta}_2 + B_{42}\dot{\eta}_2 + (A_{44} + I_4)\ddot{\eta}_4 + (B_{44} + B^*_{44})\dot{\eta}_4 + C_{44}\eta_4 + (A_{46} - I_{46})\ddot{\eta}_6 + B_{46}\dot{\eta}_6 + C_{46}\eta_6 = F_4 e^{-i\omega_e t} \tag{8}$$

$$\text{Yaw} \quad A_{62}\ddot{\eta}_2 + B_{62}\dot{\eta}_2 + (A_{64} - I_{46})\ddot{\eta}_4 + B_{64}\dot{\eta}_4 + C_{64}\eta_4$$
$$+ (A_{66} + I_6)\ddot{\eta}_6 + B_{66}\dot{\eta}_6 + C_{66}\eta_6 = F_6 e^{-i\omega_e t} \tag{9}$$

where m is the vessel's mass, Z_g is the location of the vertical center of gravity on the z axis, I_j is the moment of inertia in the jth mode, and I_{jk} is the product of inertia.

Considering the hull alone, as in the hydrofoil retracted mode, B^*_{44} in the roll equation is the nonlinear viscous damping of the hull. It is in the form of a quasi-linear function in terms of ω, viscosity, hull geometry, and the maximum roll amplitude for a given wave slope and heading, at zero speed. The second set of equations in sway, roll, and yaw may have to be solved a number of times until the difference between the maximum estimated roll angle and the computed maximum roll angle is within an acceptable tolerance. In the program the allowed tolerance is one degree.

The derivation of the hull added mass, damping and hydrostatic restoring coefficients and the exciting forces and moments used in the "DTNSRDC Ship-Motion and Sea-Load Computer Program" are presented as three-dimensional problem of ship-sections oscillating in the free surface which are determined by a close-fit source-distribution method presented in Reference 6.

With the assumed insignificance of the hull and foil system interaction, the hull and foil system contriubition to the added mass, damping and restoring coefficients and the forcing functions are simply additive. For example, the added mass coefficients A_{jk} can be expressed as

$$A_{jk} = A^H_{jk} + A^F_{jk} \tag{10}$$

where A^H_{jk} are the hull added masses and A^F_{jk} are the frequency independent foil system added masses.

HYDROFOIL EQUATIONS OF MOTION

In the derivation of the hydrofoil equations of motion three coordinate systems will be used. One system is the inertial reference frame with axes x_o, y_o, and z_o and unit vectors i_o, j_o, and k_o, respectively. The second coordinate system is fixed with respect to the hullborne hydrofoil craft such that the z = 0 plane lies in the still waterplane; z axis passes through the craft's CG; the positive x axis passes through the stern; and the positive y axis points to starboard. The unit vectors in the x, y, z axes are i, j, k, respectively. The third coordinate system, denoted by prime ('), is also fixed with respect to the body. In this system z' = 0 is the equation for the foil element with the origin (x = X, y = Y, z = Z) at its midpoint, with the axes x and x' parallel to each other. The primed system is thus translated with respect to the unprimed system by X, Y, Z and rotated about the x' axis by the dihedral angle Γ, as shown in Figure 3.

Hydrofoil Coefficients

The force on a rectangular foil element with chord c and span s at zero forward speed is given by the expression

$$a(\dot{v}_N + \dot{w}_N) + b_1(v_N + w_N)^2 + b_2 v_N \tag{11a}$$

where $a(\dot{v}_N + \dot{w}_N)$ +

a = added mass coefficient

b_1 = viscous damping coefficient

b_2 = wave-making damping coefficient

v_N = normal velocity component on the foil element due to craft motion

w_N = normal wave velocity component on the foil element

Upon linearizing the viscous damping term by the principle of equivalent energy balance which equates the energy dissipated by a non linear damped system to that of a linear damped system, equation (11a) becomes

$$a(\dot{v}_N + \dot{w}_N) + \frac{8}{3\pi} b_1 |v_N + w_N| (v_N + w_N) + b_2 v_N \tag{11b}$$

In determining the normal velocity v_N and the normal acceleration $\dot{v}_N$, one first notes that the velocity of the foil element with respect to the inertial system in vector notation is

$$\bar{v}_o = \bar{v} + (\bar{w} \times \bar{r}) \tag{12a}$$

$$= \begin{vmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \end{vmatrix} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \dot{\eta}_4 & \dot{\eta}_5 & \dot{\eta}_6 \\ x & y & z \end{vmatrix} \tag{12b}$$

With the unit normal on the element as

$$\bar{n} = -\bar{j} \sin\Gamma + \bar{k} \cos\Gamma \tag{13}$$

the normal velocity on the foil element in the body reference system becomes

$$v_N = (\bar{v}_o \cdot \bar{n})\bar{n} \tag{14a}$$

$$= -\sin\Gamma(\dot{\eta}_2 + x\dot{\eta}_6 - z\dot{\eta}_4) + \cos\Gamma(\dot{\eta}_3 + y\dot{\eta}_4 + x\dot{\eta}_5) \quad \bar{n} \tag{14b}$$

and the normal acceleration

$$\dot{v}_N = -\sin\Gamma(\ddot{\eta}_2 + x\ddot{\eta}_5 - z\ddot{\eta}_4) + \cos\Gamma(\ddot{\eta}_3 + y\ddot{\eta}_4 - x\ddot{\eta}_5) \quad \bar{n} \tag{15}$$

For harmonic oscillations with a frequency of ω, the normal velocity and acceleration can also be expressed as

$$V_N = v_N e^{i\omega t} \text{ and } \dot{v}_N = i\omega v_N e^{i\omega t}$$

where ($\hat{\ }$) denotes complex amplitude. Likewise,

$$\eta_j = \hat{\eta}_j e^{i\omega t} \quad ; \quad j = 2, \ldots, 6$$

and $\eta_j = i\omega\hat{\eta}_j\, e^{i\omega t}$

where $i = \sqrt{-1}$. It is implied that only the real part on the right-hand sides is retained, for the real time domain representation.

To obtain the normal wave velocity w_N and acceleration $\dot{w}_N$, with a wave frequency of ω, one first notes the complex orbital wave velocity potential at a point x_o, y_o, z_o in the inertial reference system to be

$$\phi^* = -i\zeta\frac{\omega}{k}\, e^{k\left[z_o - i\,(y_o \sin\mu + x_o \cos\mu)\right] + i\omega t} \tag{16}$$

where ζ is the amplitude, k is the wave number ω^2/g, and μ is the heading angle as defined and illustrated in Figure 2. The normal wave velocity on the foil element at its mean position in the body reference system is

$$\begin{aligned} w_N &= (\bar{\nabla}\phi \cdot \bar{n})\bar{n} \\ &= \zeta\omega\left[-\sin\mu\, \sin\Gamma + i\, \cos\Gamma\right] \exp\{k\left[z - i(y \sin\mu \right.\\ &\qquad \left.+\, x \cos\mu)\right] + i\omega t\}\,\bar{n} \end{aligned} \tag{17}$$

and the acceleration

$$\dot{w}_N = i\omega w_N \tag{18}$$

The expressions for v_N, $\dot{v}_N$, w_N, and $\dot{w}_N$ can now be transformed into the primed system where

$$x = X + x'$$

$$y = Y + y' \cos\Gamma$$

$$z = Z + y' \sin\Gamma$$

$$(\text{at } z' = 0)$$

With the neglect of chord-wise variations the normal velocities in the primed system become

$$v_N' = (v_N + y'\,\dot{\eta}_4)\bar{n} \tag{19a}$$

$$\dot{v}_N' = (\dot{v}_N + y'\,\ddot{\eta}_4)\bar{n} \tag{19b}$$

$$w'_N = \hat{W}_N \exp[i\omega t + ky'(\sin\Gamma - i\cos\Gamma\sin\mu)]\,\overline{n} \tag{19c}$$

$$\dot{w}_N = i\omega w'_N \tag{19d}$$

where
$$V_N = -\sin\Gamma(\dot{\eta}_2 + X\dot{\eta}_6 - Z\dot{\eta}_4) + \cos\Gamma(\dot{\eta}_3 + Y\dot{\eta}_4 - X\dot{\eta}_5)$$

$$\dot{V}_N = -\sin\Gamma(\ddot{\eta}_2 + X\ddot{\eta}_6 - Z\ddot{\eta}_4) + \cos\Gamma(\ddot{\eta}_3 + Y\ddot{\eta}_4 - X\ddot{\eta}_5)$$

$$\hat{W}_N = \zeta\omega\,(-\sin\mu\,\sin\Gamma + i\,\cos\Gamma)\cdot\exp\{k[Z - i\,(Y\sin\mu + X\cos\dot{\mu})]\}$$

Inertial Forces

The inertial forces on the foil element are now obtained by evaluating the first term *in (11b); using the accelerations* $\dot{v}'_N$ and $\dot{w}'_N$, using $0.25\pi\rho c^2$ for a, the added hydrodynamic mass for a plate of unit length (Reference 7); and integrating over the span. The resulting translational modes in surge, sway, and heave are

Surge $$A_1 = 0 \tag{21a}$$

Sway $$A_2 = Q_1(D_1\sin^2\Gamma - D_2\sin\Gamma\cos\Gamma) + Q_2\sin\Gamma \tag{21b}$$

Heave $$A_3 = Q_1(D_1\sin\Gamma\cos\Gamma - D_2\cos^2\Gamma) - Q_2\cos\Gamma \tag{21c}$$

where
$$Q_1 = 0.25\pi\rho c^2 s$$

$$D_1 = \ddot{\eta}_2 + X\ddot{\eta}_6 - Z\ddot{\eta}_4$$

$$D_2 = \ddot{\eta}_3 + Y\ddot{\eta}_4 - X\ddot{\eta}_5$$

$$Q_2 = -0.25\pi\rho c^2\zeta\omega^2\,[\cos\Gamma + i\,\sin\mu\,\sin\Gamma]\ X$$

$$\frac{\sinh[0.5ks(\sin\Gamma - i\,\sin\mu\,\cos\Gamma)]}{k(\sin\Gamma - i\,\sin\mu\,\cos\Gamma)}\ X$$

$$\cdot\exp\{k[Z - i\,(Y\sin\mu + X\cos\mu)] + i\omega t\}$$

The rotational modes in roll, pitch, and yaw can be expressed as follows

Roll $A_4 = A_2Z - A_3Y + A_4'$ (21d)

Pitch $A_5 = A_3X + A_5'$ (21e)

Yaw $A_6 = A_2X + A_6'$ (21f)

where the last terms are the hydrodynamic moments about the applicable axis passing through the element's midpoint. They are

$$A_4' = Q_1 \frac{S^2}{12} \ddot{\eta}_4$$

$$A_5' = Q_1 \frac{C^2}{32} \cos^2 \Gamma \ddot{\eta}_5$$

$$A_6' = Q_1 \frac{C^2}{32} \sin^2 \Gamma \ddot{\eta}_6$$

Viscous Damping Forces

The linearized viscous damping term in (11b) with b, evaluated for a foil element of unit length is

$$\frac{4}{3\pi} \rho C_D c \, |v_N + w_N| (v_N + w_N) \quad (22)$$

C_D is the drag coefficient for a plate with an angle of attack, α, to the direction of motion. From Reference 8

$$C_D = \begin{matrix} 0.0467\alpha & \alpha < 40° \\ 1.17 & \alpha \geq 40° \end{matrix}$$

The angle of attack α' is given for $0^0 \leq \Gamma \leq 180^0$ as

pitch and heave	$\alpha' = \lvert 90° - \Gamma \rvert$
sway and yaw	$a' = \Gamma$
roll	$a' = \lvert \arctan(-\frac{Y}{Z}) - \Gamma \rvert$
surge	$a' = 0$

wherefrom

$\alpha = \alpha'$	$0° \leq \alpha' \leq 90°$
$\alpha = 180° - \alpha'$	$90° \leq \alpha' \leq 180°$

With

$$v_N' + w_N' = (\hat{v}_N' + \hat{w}_N') \, e^{i\omega t} \quad (23)$$

The complex amplitude can be determined as

$$\hat{v}'_N + \hat{w}'_N = \hat{V}_N + i\,\omega\hat{n}_4 y' + \hat{W}_N \exp[ky'(\sin\Gamma - i\cos\Gamma\sin\mu)] \tag{24}$$

where

$$V_N = i\,\omega[\sin\Gamma\,(\hat{n}_2 + X\hat{n}_6 - Z\hat{n}_4) + \cos\Gamma(\hat{n}_3 + Y\hat{n}_4 - X\hat{n}_5)]$$

The absolute value thereof is

$$|v'_N + w'_N| + [(v'_N + w'_N)\ (v'_N + w'_N)^*]^{1/2} \tag{25}$$

where * denotes the complex conjugate. Expanding (25) in a Taylor Series (assuming it is analytic in y') about the axis y' = 0 and retaining only the first order terms result in

$$|\hat{v}'_N + \hat{w}'_N| = f(y') = H_1 + H_2 y' \tag{26}$$

where

$$H_1 = [\hat{V}_N\hat{V}^*_N + 2R\,(\hat{V}_N\hat{W}_N{}^*) + \hat{W}_N\hat{W}_N{}^*]^{1/2}$$

$$H_2 = \frac{1}{H_1} R\{(\hat{V}_N + \hat{W}_N)\ [i\,\omega\,\hat{n}_4 + \hat{W}_N k(\sin\Gamma - i\cos\Gamma\sin\mu)]^*\}$$

with R referring to the real part of a complex quantity. The viscous forces can now be determined by substituting (26) and (22) and integrating over the span, that is

$$\frac{4}{3\pi}\rho\, C_D c \int f(y')\ (v'_N + w'_N)\ dy' \tag{27}$$

In terms of the degrees of freedom, the forces are

Surge	$B_1 = 0$	(28a)
Sway	$B_2 = \frac{4}{3\pi}\rho C_D c \sin\Gamma\, P_1$	(28b)
Heave	$B_3 = \frac{4}{3\pi}\rho C_D c \cos\Gamma P_1$	(28c)
Roll	$B_4 = \frac{4}{3\pi}\rho C_D c\, P_2$	(28d)
Pitch	$B_5 = \frac{4}{3\pi}\rho C_D c\, X\cos\Gamma P_1$	(28e)
Yaw	$B_6 = \frac{4}{3\pi}\rho C_D c\, X\sin\Gamma P_1$	(28f)

where P_1 and P_2 are the following integrals

$$P_1 = \int f(y')\ (v_n' + w_n')\ dy' \tag{29}$$

$$= V_N Q_{10} + \dot{\eta}_4\ Q_{11} + \hat{W}_N e^{i\omega t} Q_{20}$$

$$P_2 = \int f(y')\ (v_N' + w_N')\ (Z\ \sin\Gamma + Y\ \cos\Gamma + y')dy'$$

$$= (Z\sin\Gamma + Y\cos\Gamma)P_1 + V_N Q_{11} + \dot{\eta}_4 Q_{12} + \hat{W}_N e^{i\omega t} Q_{21}$$

The subscripted Q's are evaluated as follows:

$$Q_{10} = \int f(y')dy' = H_1 s$$

$$Q_{11} = \int f(y')y'dy = \frac{H_2 s^3}{12}$$

$$Q_{12} = \int f(y')y'^2 dy' = \frac{H_1 s^3}{12}$$

$$Q_{20} = \int f(y')e^{Gy'}dy'$$

$$= 2(\frac{H_1}{G} - \frac{H_2}{G^2})\ \sinh(\frac{Gs}{2}) + \frac{H_2 s}{G}\ \cosh(\frac{Gs}{2})$$

$$Q_{21} = \int f(y')y'e^{Gy'}dy'$$

$$= (\frac{4H_2}{G^3} + \frac{2H_1}{G^2} + \frac{H_2 s^2}{2G})\ \sinh(\frac{Gs}{2}) + \frac{s}{G}\ (H_1 - \frac{2H_2}{G})\ \cosh(\frac{Gs}{2})$$

and $G = k(\sin\Gamma - \sin\mu\ \ \cos\Gamma)$

Wave-making Damping

A foil element intersecting or near the free surface will generate waves when oscillated laterally. Although this resitance is of lesser importance it should nonetheless be included for the zero speed case. The wave-making damping for a vertical foil element oscillating in sway is

$$\frac{\pi}{2}\ \rho\omega C_w c s^2 \dot{\eta}_2 \tag{31}$$

where C_w is a function of $\omega^2 s/g$ (Reference 5). A plot of C_w obtained by the Frank close fit method is given in Figure 4. From (34) the following damping forces have been obtained for an element with a dihedral angle of Γ

$$\text{Surge} \quad C_1 = 0 \tag{32a}$$

$$\text{Sway} \quad C_2 = \frac{\pi}{2}\rho\omega C_w cs^2 \sin\Gamma\, \dot{\eta}_2 \tag{32b}$$

$$\text{Heave} \quad C_3 = \frac{\pi}{2}\rho\omega C_w cs^2 \cos\Gamma\, \dot{\eta}_3 \tag{32c}$$

$$\text{Roll} \quad C_4 = \frac{\pi}{2}\rho\omega C_w cs^2 (Z \sin\Gamma + Y \cos\Gamma)\, \dot{\eta}_4 \tag{32d}$$

$$\text{Pitch} \quad C_5 = \frac{\pi}{2}\rho\omega C_w cs^2\; X \cos\Gamma \dot{\eta}_5 \tag{32e}$$

$$\text{Yaw} \quad C_6 = \frac{\pi}{2}\rho\omega C_w cs^2\; X \sin\Gamma \dot{\eta}_6 \tag{32f}$$

The hydrofoil related forces and moments developed and given in equation (21), (28), and (32) as A_j, B_j, C_j for the six degrees of freedom $j = 1, \ldots, 6$ are now multiplied by a correction factor for finite spans. This factor is $\frac{AR}{AR+3}$ where AR is the foil element's aspect ratio s/c. The contributions of all the individual foil elements of the complete hydrofoil system are then summed and the resulting total of A_j, B_j, and C_j separable into the motion and wave excitation terms, are added to the corresponding hull related terms in Equations 4 through 9 with the assumption that additional hull-foil interaction terms are negligible.

Further simplification was required to make the program more tractable by neglecting some foil system attributable cross coupling terms; justified on the basis that their contributions were insignificant as evident from computed and experimental results. Also simplified was the quadratic damping force by treating v_N and w_N independently whereby equation (22) becomes

$$\frac{4}{3\pi}\rho\, C_D c |v_N| v_N \text{ and } \frac{4}{3\pi}\rho\, C_D c |w_N| w_N$$

COMPUTER PROGRAM

Based on the foregoing mathematical model, the program, as presented in Reference 2, was expanded to include the hullborne hydrofoil motions in regular waves at zero speed as well as forward speed at an arbitrary heading. The basic program was the existing DTNSRDC Ship-Motion Computer Program. In itself the basic program can determine the hydrofoil craft's motions in the foil up mode in 6DOF in regular unidirectional waves of any heading. The modification consists of adding the foil system's coefficients of motion and its excitation forces and moments to the corresponding terms for the hull. As a consequence, three card sets listed below and pertaining to the foil system of the hullborne hydrofoil craft are added onto the existing 34 data card sets.

A. Input Description

Data Card Set 35 - one card with format (3I4)

IFOIL: 2 for hydrofoil craft in the foils down mode. All other integer values are for retracted foil systems where only the hull is subjected to hydrodynamic forces.

NZERO: number of zero speed cases for the hullborne hydrofoil in the foils down mode

IPRINT: option of printing the matrix equations of motion. With IPRINT = 0 printing of matrices is suppressed and for IPRINT = 1 printing of matrices takes place.

Data Card Set 36 - One card with format (I5, 3F12.2)

NF: total number of foil elements on the starboard side of the hydrofoil craft. This total consists of the elements in symmetry about the xz-plane plus the elements lying in the xz-plane.

FVOL: is the displaced volume of the entire foil system (including the portions of the struts that are immersed). The unit is WORD**3, (see Data Card Set 4 of Reference 1)

FXCB: the foil system's longitudinal center of buoyancy, LCB, with respect to the entire craft's LCB, i.e. the x value in the body coordinate system in units of WORD.

FZCB: the foil system's vertical center of buoyancy, VCB, i.e. the z value in the body coordinate system in units of WORD.

Data Card Set 37 - one card per foil element with format (F3.0, 5F7.2, F5.0, F10.7, F5.0, F5.0)

CPL: If the plane foil element lies in the center plane, i.e the xz-plane,

CPL = 1 and in all other cases

CPL = 2 where symmetry of the elements about the center plane is assumed.

The remaining parameters in this set pertain to the foil element or portion thereof on the starboard side of the centerplane or for the element lying in the centerplane.

SPAN: the length of the rectangular foil element taken in a line parallel to the yz-plane in units of WORD.

CHORD: the width of the rectangular foil element taken in a line parallel to the xz-plane in units of WORD.

X: x coordinate of the foil element's midpoint in units of WORD

Y: y coordinate of the foil element's midpoint in units of WORD

Z: z coordinate of the foil element's midpoint in units of WORD

DGAMMA: is the dihedral angle, i.e. the angle between the foil element and the y-axis. If the foil lies in the centerplane DGAMMA = 90. The range of DGAMMA is between 0 and 180 degrees.

CLZ: is the vertical lift slope of the foil element in dimensionless units (not used in zero speed case)

ASP: is a positive number utilized in the aspect ratio correction factor AR/(AR + ASP) for foil elements of finite span. For the example cited ASP = 3.

STRUT: 0. for a non-surface piercing foil element. All other values are for surface piercing foils. This control variable for the wave-generated damping is used in the zero speed case only.

Data Card Set 38 - NZERO cards, one for each hullborne hydrofoil zero speed case foils down mode. The format is (2F8.2, 2F8.4)

AMPMOT(2): Estimated maximum sway amplitude in units of WORD

AMPMOT(3): Estimated maximum heave amplitude in units of WORD

AMPMOT(5): Estimated maximum pitch amplitude in radians

AMPMOT(6): Estimated maximum yaw amplitude in radians

The program for the zero speed case uses a "trial and error" procedure for solving the quasi-linear set of equations in sway, roll, and yaw. Keeping the waveslope constant, the initial maximum sway, roll, and yaw amplitudes are used in computing the corresponding motion responses. The program then compares the maximum computed amplitudes with the estimated values and if the differences in sway or yaw is greater than 8 percent or the difference

in roll is greater than 1 degree new estimates are determined where needed and used in the repeated computation for motion responses. These iterations take place until the computed and estimated maximum amplitudes in sway, roll, and yaw fall within the acceptable limits. If the proper closure still does not occur for the fifth iteration, the program prints the results for the last iteration which includes a table of the estimated and computed values and the differences and a note as to the response where the proper closure failed. Although it is advisable to have good estimates in maximum pitch and heave amplitudes, errors in these are not as critical. As a consequence the program does not determine new estimates for pitch and heave.

The program's output is essentially the same as for the original DTNSRDC Ship-Motion Computer Program (Ref.1). In addition, a listing of the foil related input cards, Data Card Sets 35 through 38, are given as well as the foil related non-dimensionalized coefficients and forcing functions and a modified listing of the estimated and computed maximum response amplitudes to include sway and yaw as well as roll. As an option the matrix equations for the hull alone, for the hydrofoil system, for the combined hull and foil system, and the solved simultaneous differential equations with the inverted matrix can be printed.

In addition to the original subroutines of the DTNSRDC Ship-Motion are the subroutines FOIL, THEO, and EXCIT for the hullborne hydrofoil, foils down mode, at speeds greater than zero and the subroutine ZERO for the zero speed case. Appendix A presents the update cards needed for the original DTNSRDC Ship-Motion Computer Program and Appendix B presents the subroutines ZERO, FOIL, THEO, and EXCIT. The latter two subroutines require a subprogram for computing Bessel functions. The entire program is on digital 7-track magnetic tape which can be used on the CDC 6000 computer system.

COMPARISON OF PREDICTED AND EXPERIMENTAL RESULTS

The first experimental results chosen for comparison with predicted results were the hullborne motion measurements of the 313-ton Plainview AG(EH)-1 hydrofoil craft. The experiments were conducted by R.M. Vuolo on a 1:12 scale model in DTNSRDC's Harold E. Saunders Seakeeping and Maneuvering Facility. The model was run in both the foils up and foils down modes in unidirectional regular waves. The full scale speeds were 6 and 12 knots at the three headings of head (180^0), bow (150^0), and beam waves (90^0). The regular waves were of constant wave steepness 1/60, and wave lengths ranged from $L/\lambda = 0.33$ to $L/\lambda = 4.0$, corresponding to wave frequencies of $\omega = 0.57$ to 1.98 rad/sec.

The Hullborne Hydrofoil Six-Degree-of-Freedom Motion Program was likewise run in both the foils up and foils down modes at the three headings of 180^0, 150^0 and 90^0. The predicted motions at craft speeds of 6 and 12 knots are presented and compared with the experimental results in an earlier report, Reference 2. At zero speed the predicted motions at the three headings generally agreed well with the experimental results as givne in Figures 5a, 5b, and 5c which show the craft's transfer functions in heave, roll, and pitch versus wave frequency together with the phase lag with respect to the maximum height of the wave at the CG. The effect of the immersed hydrofoil system on pitch and heave is insignificant at all three headings both theoretically and experimentally. Roll is the only degree of freedom notably affected by the immersion of the hydrofoil system as seen in the 90 and 150 degree headings, Figures 5b and 5c. Some of the minor discrepancies between the predicted and experimenal results, especially in roll, are attributable most likely to inaccurate roll

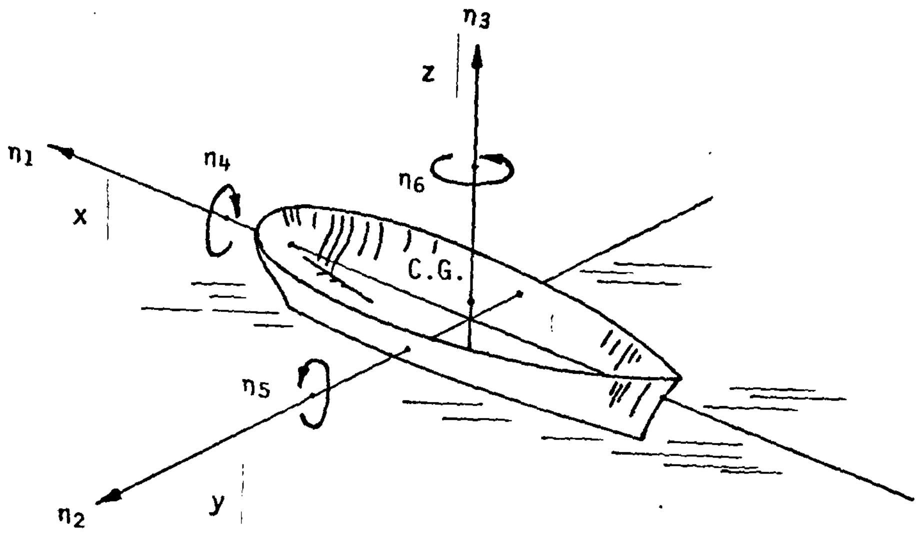
gyradii estimates since the model's roll gyradii for the foil up and the foil down modes were not available. A less significant source of error may also be inaccuracies in the foil system's displacement and center of buoyancy estimates.

The second set of experimental results used for comparison with the predicted motions of the Hullborne Hydrofoil 6DOF Motion Prediction Program was a 1:20 scale model of the 220 L-TON PHM hydrofoil craft. The mdoel was tested at the Davidson Laboratory in simulated beam Sea States 3 and 5 in the foils submerged, hullborne model at zero speed, Reference 9. Shown in Figure 6 are the measured roll responses in model scale together with predicted roll responses which agree satisfactorily.

CONCLUDING REMARKS

The motions predicted for the AG(EH)-1 hydrofoil craft at zero speed in the hullborne model with retracted and submerged hydrofoils are in satisfactory agreement with the available expeirmental data for regular waves at 90, 150 and 180 degrees headings. The DTNSRDC Hullborne Hydrofoil 6DOF Motion Prediction Computer Program also predicted satisfactorily the roll response at zero speed of the PHM hydrofoil craft in beam Sea States 3 and 5 as compared to experimental results.

Additional comparisons directed toward verifications of the DTNSRDC Hullborne Hydrofoil 6DOF Motion Prediction Computer Program should be made as more experimental results become available. Although the program can be used at present to predict craft motions at zero and forward speeds, no verification has been made of craft responses at wave headings less than 90 degrees due to a lack of experimental data.



TRANSLATORY DISPLACEMENTS

η_1 = SURGE
η_2 = SWAY
η_3 = HEAVE

ANGULAR DISPLACEMENTS

η_4 = ROLL
η_5 = PITCH
η_6 = YAW

Figure 1 - Sign Convention of Body Coordinate System

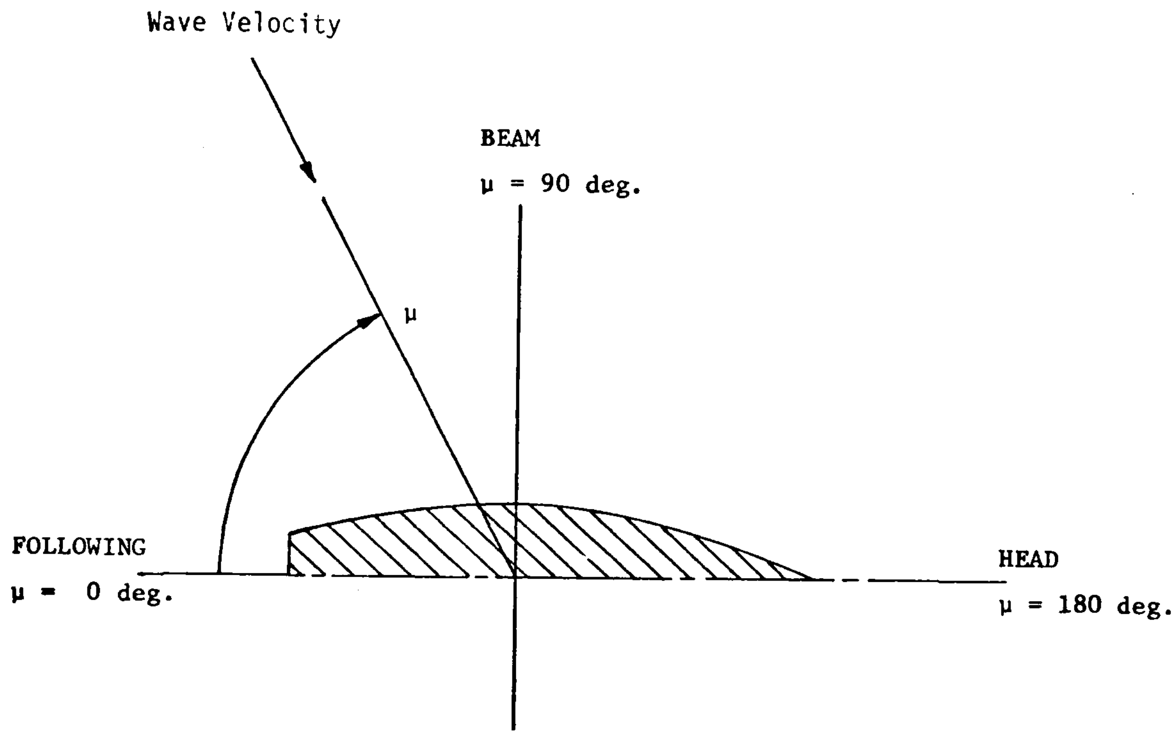


Figure 2 - Definition of Heading Angle, μ

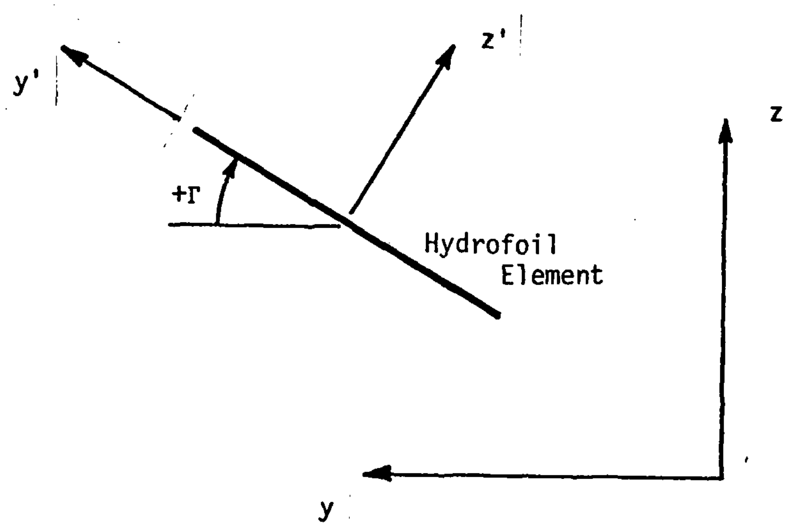
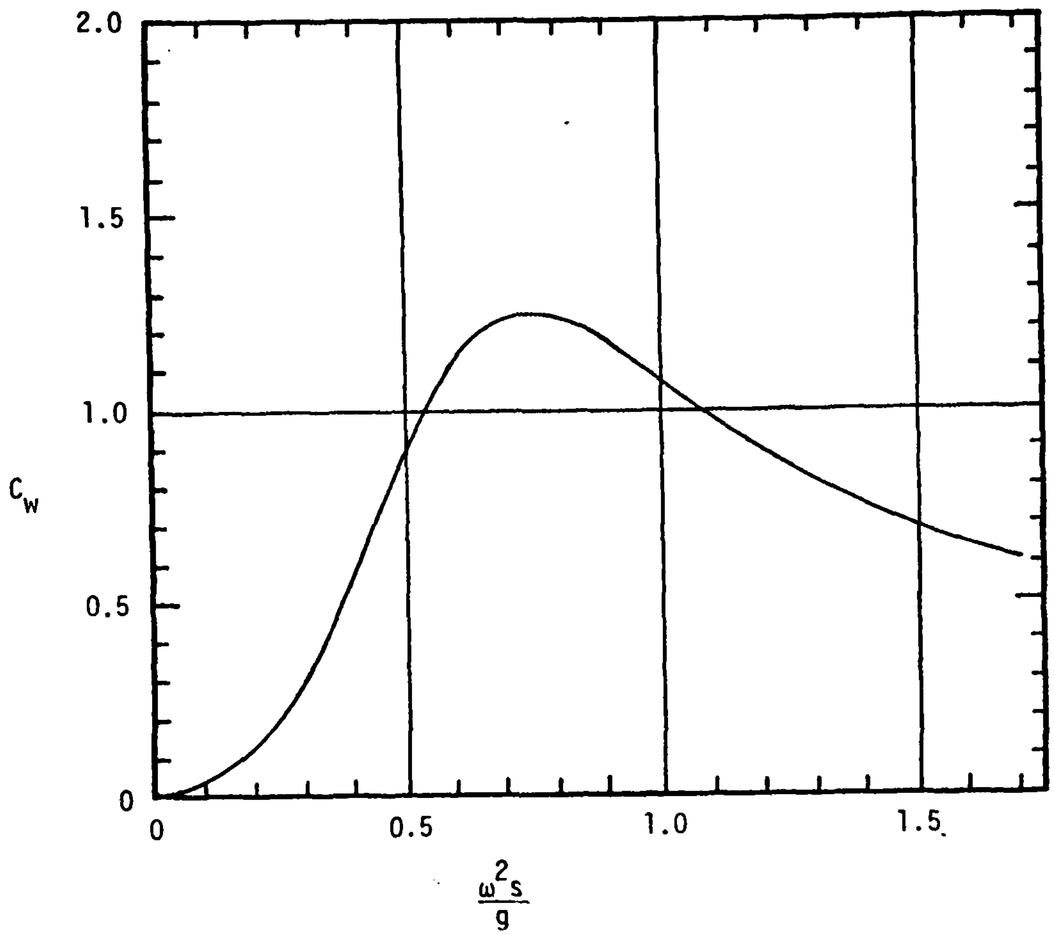


Figure 3 - Primed Coordinate System and Dihedral Angle, Γ, in xyz Body Coordinate System



$\left(\begin{array}{ll} \omega & \text{circular wave frequency} \\ s & \text{span} \\ g & \text{gravitational acceleration} \end{array} \right)$

Figure 4 - Strut Wave-Making Damping Coefficient, C_w

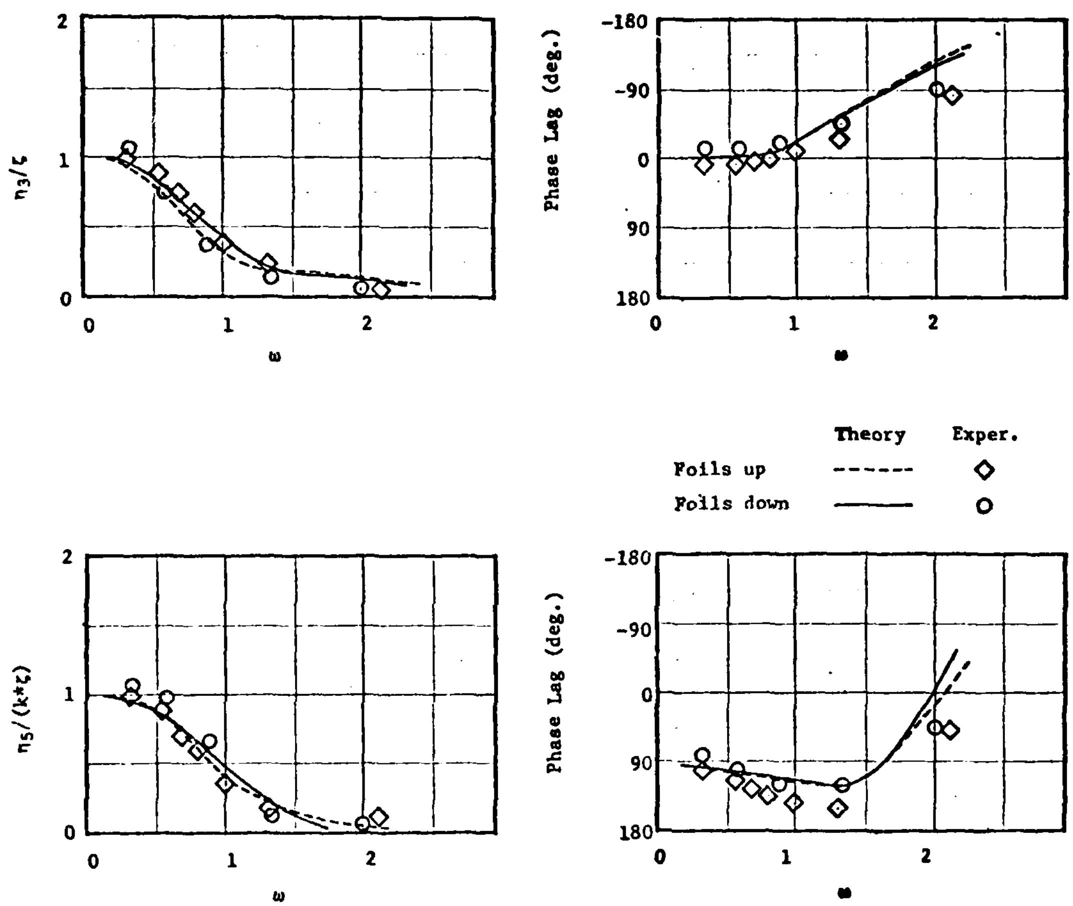


Figure 5a -- Non-Dimensional Transfer Function and Phase Versus ω of the AG(EH) 1 in Head Seas, $\mu=180^{\circ}$

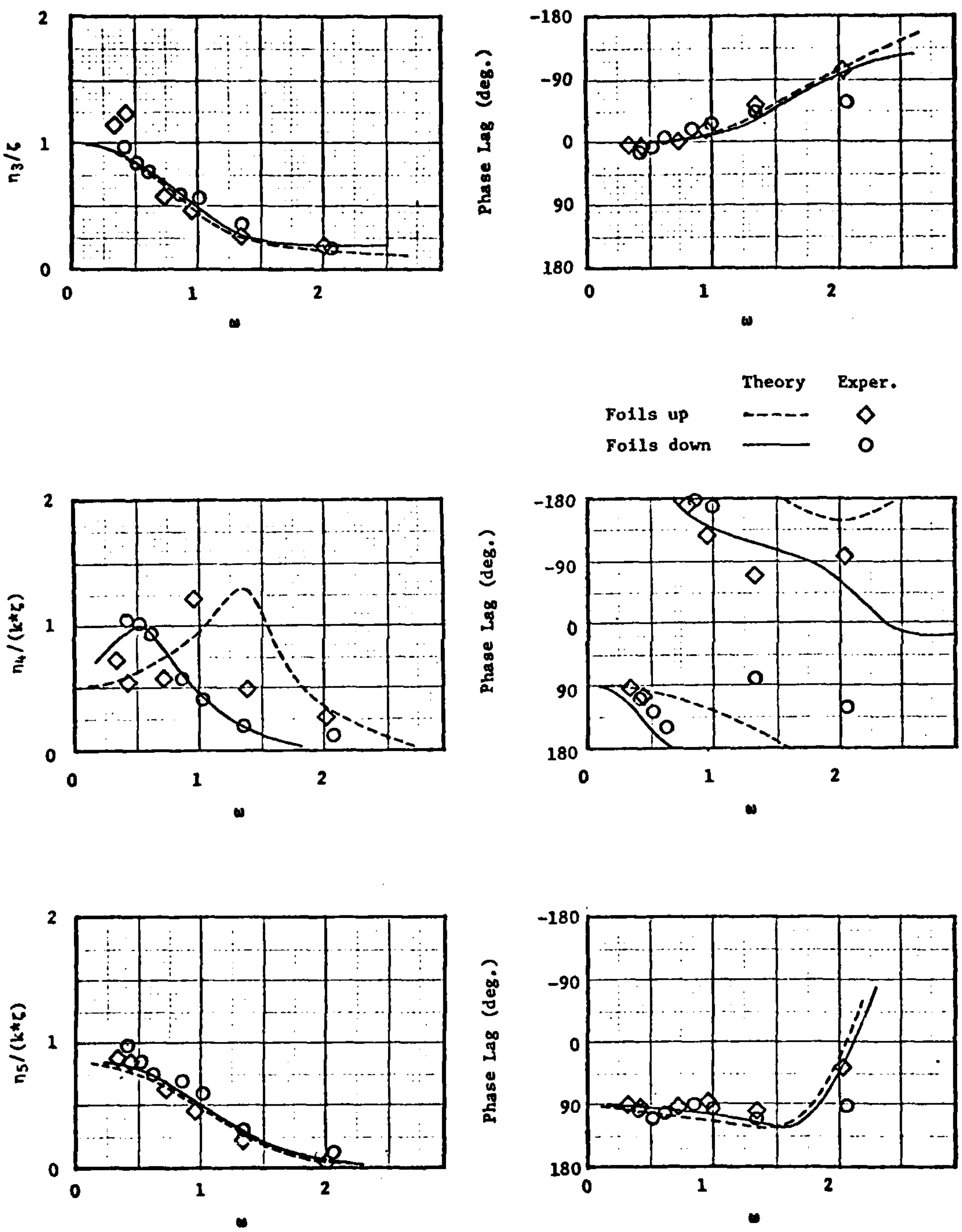


Figure 5b - Non-Dimensional Transfer Function and Phase Versus ω of the AG(EH)-1 in Bow Seas, $\mu=150^o$

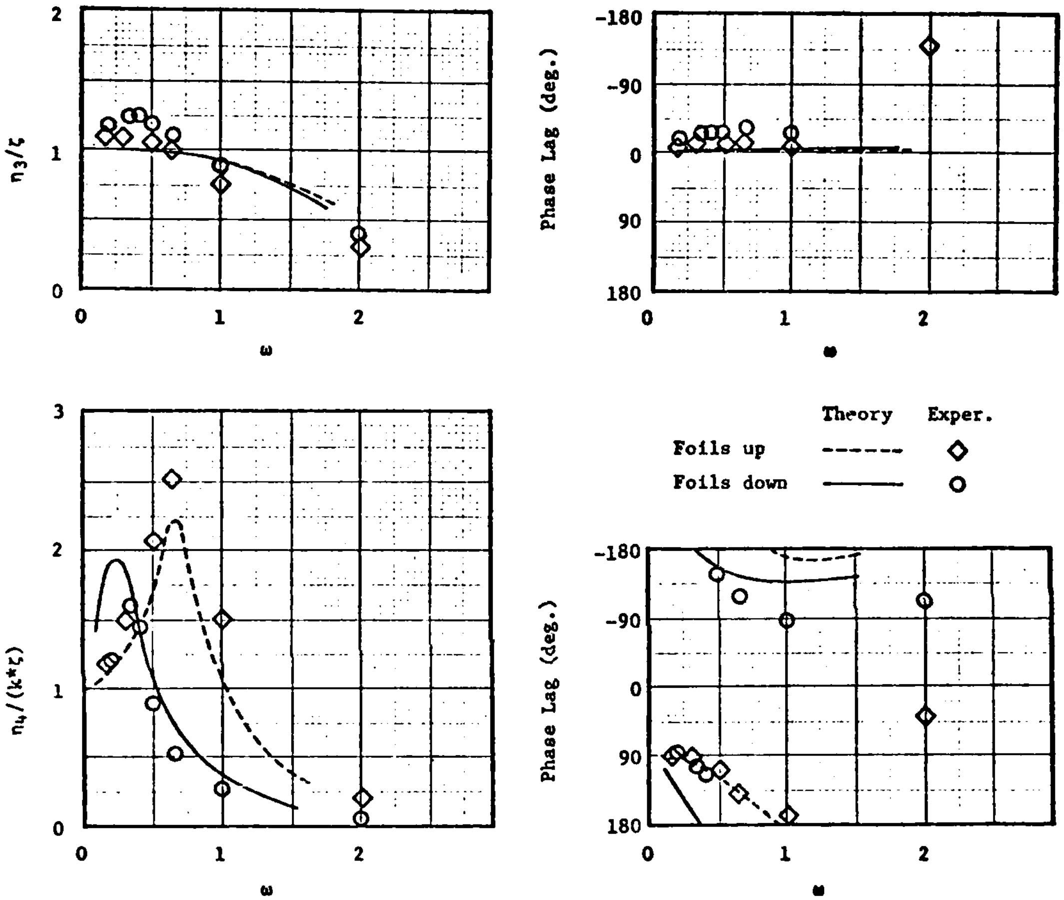


Figure 5c - Non-Dimensional Transfer Function and Phase Versus ω of the AG(EH)-1 in Beam Seas, $\mu=90^{\circ}$

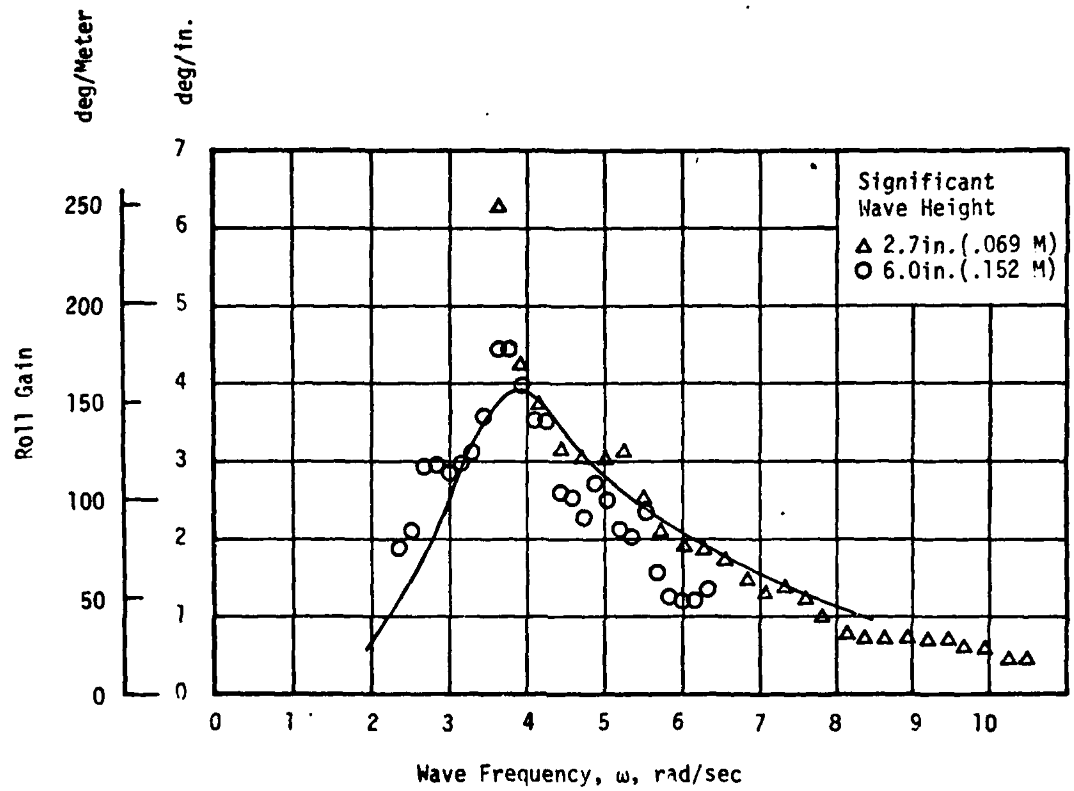


Figure 6 - Roll Gain versus Wave Frequency of the PHM Hydrofoil Craft in Beam Sea States 3 and 5

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3. Salvesen, N., E.D. Tuck, and O. Faltinsen, "Ship Motions and Sea Loads," Transactions of the Society of Naval Architects and Marine Engineers, Vol. 78, pp 250-287 (1970).

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APPENDIX A

LIST OF UPDATE CARDS FOR DTNSRDC 6DOF SHIP-MOTION PREDICTION COMPUTER PROGRAM

The update cards listed below are required to modify the existing DTNSRDC 6DOF Ship-Motion and Sea-Load Computer Program to the Hullborne Hydrofoil 6DOF Motion Prediction Computer Program. This modification allows the use of the hydrofoil related subroutines presented in Appendix B.

```
*ID FMOD
*INSERT LK0.25
      COMMON /PFOIL/ DUM5(165)
      COMMON /QFOIL/ DUM6(36)
*INSERT PR1.22
      COMMON /PFOIL/ RHO,IFOIL,NZERO,IPRINT,NF,Q(10,10),AMPMOT(6,10)
*D PR1.255
      IF(IDAMP-2) 9090,9055,9080
*INSERT PR1.280
  905 FORMAT(     I5,3F12.2)
  906 FORMAT(12X,I5,3F12.2)
  907 FORMAT(     I5,3F12.2)
  970 FORMAT(     F3.0,5F7.2,F5.0,F10.7,F5.1,F5.0)
  971 FORMAT(12X,F3.0,5F7.2,F5.0,F10.7,F5.1,F5.0)
  980 FORMAT(//13H  CARD SET 36)
  990 FORMAT(//13H  CARD SET 37)
 7001 FORMAT(2I4)
 7002 FORMAT(//22H  FOIL DATA CARD INPUT)
 7003 FORMAT(//32H1 HYDROFOIL VESSEL WITH FOILS UP/)
 7004 FORMAT(//34H1 HYDROFOIL VESSEL WITH FOILS DOWN/)
 2212 FORMAT (/21X,1H1,9X,1H2,9X,1H3,9X,1H4,9X,1H5,9X,1H6,9X,1H7,9X,1H8/
     24X,8HCOLUMNS ,8(10H1234567890))
 2213 FORMAT(/14X,2HNF,9X,4HFVOL,8X,4HFXCB,8X,4HFZCB/)
 2214 FORMAT (/12X,3HCPL,3X,4HSPAN,2X,5HCHORD,3X,4HX(S),4X,1HY,6X,1HZ,2X
```


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```
     2,6HDGAMMA,3X,3HCLZ,5X,3HASP/)
C-----------------------------------------------------------------------
C     DATA CARD SET 35
C-----------------------------------------------------------------------
C     HYDROFOIL VESSEL WITH FOILS UP    -  IFOIL=1
C     HYDROFOIL VESSEL WITH FOILS DOWN -  IFOIL=2
C     PRINTOUT OF MATRIX EQUATIONS  (NO = 0 , YES = 1)
C
      READ(5,7001) IFOIL,IPRINT
      IF(IFOIL .NE. 2) IFOIL=1
      WRITE(1,7001) IFOIL
      IF(IFOIL-1) 9091,9091,9092
 9091 WRITE(6,7003)
      GO TO 9515
 9092 WRITE(6,7004)
      WRITE(6,7002)
      WRITE(6,2212)
C-----------------------------------------------------------------------
C     DATA CARD SET 36
C-----------------------------------------------------------------------
C     NUMBER OF INPUT FOIL ELEMENTS, DISPLACED VOLUME (WORD**3),
C     LONGITUDINAL CENTER OF BOYANCY FROM F.P. AND VERTICAL CENTER OF
C     BOYANCY FROM WATERLINE (WORD) OF THE ENTIRE HYDROFOIL SYSTEM
C
      READ(5,905) NF,FVOL,FXCB,FZCB
      WRITE(6,980)
      WRITE(6,2213)
      WRITE(6,906) NF,FVOL,FXCB,FZCB
      WRITE(1,907) NF,FVOL,FXCB,FZCB
      WRITE(6,990)
      WRITE(6,2214)
      DO 100 I=1,NF
C-----------------------------------------------------------------------
C     DATA CARD SET 37
C-----------------------------------------------------------------------
C     FOIL ELEMENT IN VERTICAL CENTER PLANE (CPL=1. FOR YES, CPL=2. FOR
C     NO) , HYDROFOIL ELEMENT SPAN (FT), CHORD (FT), COORDINATES X,Y,Z
C     OF MIDPOINT (FT), DIHEDRAL ANGLE OF V-FOIL (DEG), VERTICAL LIFT
C     SLOPE (WORD2/WORD), ASP IN THE FACTOR   AR/(AR+ASP)   FOR FINITE
C     SPAN
C
      READ(5,970)  (Q(N,I),N=1,10)
      WRITE(6,971) (Q(N,I),N=1,10)
      WRITE(1,970) (Q(N,I),N=1,10)
  100 CONTINUE
      WRITE(6,2212)
 9515 CONTINUE
*INSERT PR1.418
      IF(IFOIL-1) 51,51,52
   52 FXCB=FXCB/EL
      FZCB=FZCB/ELL
      FVOL=FVOL/EL3
      HVOL=TVOL
      TVOL=HVOL+FVOL
```

```
      TPST=(TPST*HVOL+FXCB*FVOL)/TVOL
      CBV=(CBV*HVOL+FZCB*FVOL)/TVOL
   51 CONTINUE
      RHO=THAS/(TVOL*EL3)
*I SP5.21
      COMMON /PFOIL/ RHO,IFOIL,NZERO,IPRINT,NF,Q(10,10),AMPMOT(6,10)
      COMMON /QFOIL/ GA(6,6)
*D SP5.41
      DIMENSION GMU(6,6),TODF(6,6),TEVF(6,6),BODF(6),BEVF(6),T(32,62)
      DIMENSION TODA(6,6),TEVA(6,6),BODA(6),BEVA(6),TODB(6,6),TEVB(6,6),
     2BODB(6),BEVB(6),TODC(6,6),TEVC(6,6),BODC(6),BEVC(6)
*I SP5.42
      BACKSPACE 1
 2198 FORMAT(118H1NON-DIMENSIONAL ADDED MASS,DAMPING,AND RESTORING COEFF
     2ICIENTS AND EXCITING FORCES AND MOMENTS OF THE STRUTS AND FOILS)
 2199 FORMAT(41X,12A6,15X,3H***///17X,9HHEADING =,F5.0,4H DEG,7X,12HSHIP
     2 SPEED =,F6.2,6H KNOTS/18X,16H(HEAD SEAS =180),9X,15HFROUDE NUMBER
     3 =,F7.4,///)
 2200 FORMAT(/58H NON-DIMENSIONALIZED ADDED MASS COEFFICIENTS OF THE FOI
     1LS-)
 2201 FORMAT(3X,6HWE(ND),5X,6HA(1,1),6X,6HA(2,2),6X,5HA(3,3),6X,6HA(4,4)
     1,6X,6HA(5,5),6X,6HA(6,6),6X,6HA(3,5),5X,6HA(2,6),6X,6HA(2,4),6X,6H
     2A(4,6))
 2202 FORMAT(3X,F6.3,1P10E12.4)
 2203 FORMAT(9X,1P10E12.4)
 2204 FORMAT(/55H NON-DIMENSIONALIZED DAMPING COEFFICIENTS OF THE FOILS-
     1)
 2205 FORMAT(3X,6HWE(ND),5X,6HB(1,1),6X,6HB(2,2),6X,6HB(3,3),6X,6HB(4,4)
     1,6X,6HB(5,5),5X,6HB(6,6),6X,6HB(3,5),5X,6HB(2,6),6X,6HB(2,4),6X,6H
     2B(4,6))
 2206 FORMAT(/57H NON-DIMENSIONALIZED RESTORING COEFFICIENTS OF THE FOIL
     1S-)
 2207 FORMAT(3X,6HWE(ND),5X,6HC(1,1),6X,6HC(2,2),6X,5HC(3,3),6X,6HC(4,4)
     1,6X,6HC(5,5),6X,6HC(6,6),6X,6HC(3,5),6X,6HC(2,6),6X,6HC(2,4),6X,6H
     2C(4,6))
 2208 FORMAT(/,* NON-DIMENSIONALIZED FORCE AND MOMENT FUNCTIONS OF THE F
     1OILS-*)
 2209 FORMAT(3X,6HWE(ND),4X,5HSURGE,8X,4HSWAY,7X,5HHEAVE,8X,4HROLL,7X,5H
     2PITCH,9X,3HYAW)
 2210 FORMAT(3X,F6.3,1P6E12.4)
 2211 FORMAT(9X,1P6E12.4)
 8001 FORMAT(///51H   TOD*X=BOD  BEFORE INSERTION OF HYDROFOIL ELEMENTS/)
 8011 FORMAT(///51H   TEV*X=BEV  BEFORE INSERTION OF HYDROFOIL ELEMENTS/)
 8003 FORMAT(///50H   TOD*X=BOD  AFTER INSERTION OF HYDROFOIL ELEMENTS/)
 8013 FORMAT(///50H   TEV*X=BEV  AFTER INSERTION OF HYDROFOIL ELEMENTS/)
 8002 FORMAT(///*  MATRICIES TODF AND BODF*/)
 8012 FORMAT(///*  MATRICIES TEVF AND BEVF*/)
 8021 FORMAT(9X,1P6E12.3,7X,1HX,I2,7H   REAL,8X,1P1E12.3)
 8022 FORMAT(9X,1P5E12.3,7X,1HX,I2,7H   IMAG,8X,1P1E12.3)
 8004 FORMAT(///14H  X=INVTOD*BOD/)
 8014 FORMAT(///14H  X=INVTEV*BEV/)
 8023 FORMAT(9X,1P1E12.3,12X,1P6E12.3,12X,1P1E12.3)
*D SP5.225
      IF(IBILGE-1) 3003,3003,3004
```

```
*D SP5.266
      IF(IEND-1) 3001,3001,3002
*I SP5.306
C     *     *     *     *     *     *     *     *     *     *     *     *
C     FOR A HULLBORNE HYDROFOIL (IFOIL=2), SUBROUTINE -FOIL- CALCULATES
C     THE MOTION COEFFICIENTS AND THE EXCITATION FORCES AND MOMENTS DUE
C     TO THE FOILS. RETURNED ARE THE TERMS FOR THE TOD, BOD, TEV, AND
C     BEV MATRICIES
C
      IF(IFOIL-1) 7200,7200,7100
 7100 WAMPL2=WAVAMP(LL)
      HDG2=HDG1(MM)
 7104 CALL FOIL(TODF,TEVF,BODF,BEVF,VKNOTS,WAMPL2,HDG2,GXI,ELL,RHO,NF,Q,
     2TMAS,T,LL,HVLNTH)
 7106 DO 7110 JA=1,6
      DO 7112 JB=1,6
      TODA(JA,JB)=TOD(JA,JB)
      TEVA(JA,JB)=TEV(JA,JB)
      TOD(JA,JB)=TOD(JA,JB)+TODF(JA,JB)
      TEV(JA,JB)=TEV(JA,JB)+TEVF(JA,JB)
      TODB(JA,JB)=TOD(JA,JB)
      TEVB(JA,JB)=TEV(JA,JB)
 7112 CONTINUE
 7110 CONTINUE
 7200 CONTINUE
C     *     *     *     *     *     *     *     *     *     *     *     *
*I SP5.418
C     *     *     *     *     *     *     *     *     *     *     *     *
      IF(IFOIL-1) 7600,7600,7500
 7500 DO 7510 JA=1,6
      BODA(JA)=BOD(JA,1)
      BEVA(JA)=BEV(JA,1)
      BOD(JA,1)=BOD(JA,1)+BODF(JA)
      BEV(JA,1)=BEV(JA,1)+BEVF(JA)
      BODB(JA)=BOD(JA,1)
      BEVB(JA)=BEV(JA,1)
 7510 CONTINUE
 7600 CONTINUE
C     *     *     *     *     *     *     *     *     *     *     *     *
*D SP5.435
      DO 8201 IQ=1,6
      BODC(IQ)=BOD(IQ,1)
      DO 8202 JQ=1,6
 8202 TODC(IQ,JQ)=TOD(IQ,JQ)
 8201 CONTINUE
      IF(ID-1) 501,501,502
*D SP5.437
      DO 8203 IQ=1,6
      BEVC(IQ)=BEV(IQ,1)
      DO 8204 JQ=1,6
 8204 TEVC(IQ,JQ)=TEV(IQ,JQ)
 8203 CONTINUE.
      IF(ID-1) 503,503,502
```

```
*D SP5.461
      IF(IPRES-1) 5202,5202,5203
*D SP5.490
      IF(JS-1) 6222,6222,6223
*D SP5.531
 1505 IF(IFOIL-1) 2401,2401,2402
 2402 WRITE(6,2198)
      DO 5614 JH=1,2
      IF(JH .EQ. 1) H=1
      IF(JH .EQ. 2) H=6
      IF(H .EQ. 6 .AND. PRNTOP .EQ. MIN) GO TO 5614
      WRITE(H,2199) TITO,HDIG1,VKNOTS,FN(JJ)
 5614 CONTINUE
      WRITE(6,2200)
      WRITE(6,2201)
      DO 2300 LX=1,NOK
      LWEINC=NOK-LX+1
 2300 WRITE(6,2202)    ZN(LWEINC),(T(LWEINC,KX),KX=1,10)
      WRITE(6,2204)
      WRITE(6,2205)
      DO 2301 LX=1,NOK
      LWEINC=NOK-LX+1
      WRITE(6,2202)    ZN(LWEINC),(T(LWEINC,KX),KX=11,20)
 2301 WRITE(6,2203) (T(LWEINC,KX),KX=21,30)
      WRITE(6,2206)
      WRITE(6,2207)
      DO 2302 LX=1,NOK
      LWEINC=NOK-LX+1
      WRITE(6,2202)    ZN(LWEINC),(T(LWEINC,KX),KX=31,40)
 2302 WRITE(6,2203) (T(LWEINC,KX),KX=41,50)
      WRITE(6,2208)
      WRITE(6,2209)
      DO 2303 LX=1,NOK
      LWEINC=NOK-LX+1
      WRITE(6,2210)    ZN(LWEINC),(T(LWEINC,KX),KX=51,56)
 2303 WRITE(6,2211) (T(LWEINC,KX),KX=57,62)
      IF(IPRINT) 2401,2401,8000
 8000 WRITE(6,8001)
      DO 8101 IQ=1,3
      JQ=IQ+IQ-1
 8101 WRITE(6,8021) (TODA(IQ,KQ),KQ=1,6),JQ,BODA(IQ)
      DO 8102 IQ=4,6
      JQ=IQ+IQ-7
 8102 WRITE(6,8022) (TODA(IQ,KQ),KQ=1,6),JQ,BODA(IQ)
      WRITE(6,8002)
      DO 8103 IQ=1,3
      JQ=IQ+IQ-1
 8103 WRITE(6,8021) (TODF(IQ,KQ),KQ=1,6),JQ,BODF(IQ)
      DO 8104 IQ=4,6
      JQ=IQ+IQ-7
 8104 WRITE(6,8022) (TODF(IQ,KQ),KQ=1,6),JQ,BODF(IQ)
      WRITE(6,8003)
      DO 8105 IQ=1,3
      JQ=IQ+IQ-1
```

```
 8105 WRITE(6,8021) (TODB(IQ,KQ),KQ=1,6),JQ,BODB(IQ)
      DO 8106 IQ=4,6
      JQ=IQ+IQ-7
 8106 WRITE(6,8022) (TODB(IQ,KQ),KQ=1,6),JQ,BODB(IQ)
      WRITE(6,8004)
      DO 8107 IQ=1,6
 8107 WRITE(6,8023) BODC(IQ),(TODC(IQ,KQ),KQ=1,6),BODB(IQ)
      WRITE(6,8011)
      DO 8108 IQ=1,3
      JQ=IQ+IQ
 8108 WRITE(6,8021) (TEVA(IQ,KQ),KQ=1,6),JQ,BEVA(IQ)
      DO 8109 IQ=4,6
      JQ=IQ+IQ-6
 8109 WRITE(6,8022) (TEVA(IQ,KQ),KQ=1,6),JQ,BEVA(IQ)
      WRITE(6,8012)
      DO 8110 IQ=1,3
      JQ=IQ+IQ
 8110 WRITE(6,8021) (TEVF(IQ,KQ),KQ=1,6),JQ,BEVF(IQ)
      DO 8111 IQ=4,6
      JQ=IQ+IQ-6
 8111 WRITE(6,8022) (TEVF(IQ,KQ),KQ=1,6),JQ,BEVF(IQ)
      WRITE(6,8013)
      DO 8112 IQ=1,3
      JQ=IQ+IQ
 8112 WRITE(6,8021) (TEVB(IQ,KQ),KQ=1,6),JQ,BEVB(IQ)
      DO 8113 IQ=4,6
      JQ=IQ+IQ-6
 8113 WRITE(6,8022) (TEVB(IQ,KQ),KQ=1,6),JQ,BEVB(IQ)
      WRITE(6,8014)
      DO 8114 IQ=1,6
 8114 WRITE(6,8023) BEVC(IQ),(TEVC(IQ,KQ),KQ=1,6),BEVB(IQ)
 2401  IF (IWSTP .EQ. 1) CALL EXCFM
*ID M163
*D FMOD.30
*D FMOD.48
*D FMOD.63
*D FMOD.82
*D FMOD.175,FMOD.179
      IF(PRNTOP .EQ. MIN) GO TO 5614
      WRITE(6,2199) TITO,HDIG1,VKNOTS,FN(JJ)
*ID ZMOD
*D FMOD.8,FMOD.9
  970 FORMAT(     F3.0,5F7.2,F5.0,F10.7,F5.1,F5.0)
  971 FORMAT(12X,F3.0,5F7.2,F5.0,F10.7,F5.1,F5.0)
  972 FORMAT(     4F8.2)
  973 FORMAT(12X,4F8.2)
*D FMOD.12
 7001 FORMAT(3I4)
*D FMOD.20
     2,6HDGAMMA,3X,3HCLZ,5X,3HASP,2X,3HNCH/)
 2215 FORMAT(/15X,4HSWAY,4X,5HHEAVE,3X,5HPITCH,4X,3HYAW,4X,20HESTIMATED
     2AMPLITUDES/)
*D FMOD.24,FMOD.28
C    HYDROFOIL VESSEL WITH FOIL POSITION  -  UP=1, DOWN=2
```

```
C     NUMBER OF ZERO KNOT CONDITIONS
C     PRINTOUT OF MATRIX EQUATIONS  -  NO=0, YES=1
      READ(5,7001) IFOIL,NZERO,IPRINT
*D FMOD.59
C     SPAN, HYDROFOIL PIERCING SURFACE  -  NO=0.  YES=1.
*D FMOD.65
      IF(NZERO .EQ. 0) GO TO 108
      WRITE(6,2215)
C-----------------------------------------------------------------------
C     DATA CARD SET 38
C-----------------------------------------------------------------------
C     ESTIMATED MOTION AMPLITUDES FOR ZERO SPEED CONDITIONS GIVEN BY
C     VARYING THE WAVE SLOPE FIRST FOLLOWED BY THE HEADING ANGLE. ORDER
C     OF AMPLITUDES READ IN ARE SWAY(2), HEAVE(3), PITCH(5), YAW(6)
C     (DEG,WORD)
C
      DO 101 I=1,NZERO
      READ(5,972)  AMPMOT(2,I),AMPMOT(3,I),AMPMOT(5,I),AMPMOT(6,I)
  101 WRITE(6,973) AMPMOT(2,I),AMPMOT(3,I),AMPMOT(5,I),AMPMOT(6,I)
  108 WRITE(6,2212)
*D FMOD.81
     2BODB(6),BEVB(6),TODC(6,6),TEVC(6,6),BODC(6),BEVC(6),AMPL(6)
*I SP5.59
      NZ=0
*D SP5.129
      IF(IFOIL-1) 401,401,400
  400 IF(VKNOTS .EQ. 0.) NZ=NZ+1
  401 DO 999 IWSTP=1,NWSTP
*D FMOD.128
C     BEV MATRICIES. SUBROUTINE -ZERO- IS USED FOR THE ZERO SPEED CASE.
*I FMOD.132
      IF(VKNOTS) 7200,7102,7104
 7102 AMPMOT(4,NZ)=THM
      DO 7103 JA=2,6
 7103 AMPL(JA)=AMPMOT(JA,NZ)
      CALL ZERO(TODF,TEVF,BODF,BEVF,GXI,ELL,RHO,NF,Q,TMAS,T,LL,AMPL)
      GO TO 7106
```

APPENDIX B

LISTING OF SUBROUTINES ZERO, FOIL, THEO, and EXCIT

The following subroutines are used to calculate the motion coefficients and wave excitation forcing functions of the submerged hydrofoil system of a hullborne hydrofoil craft. Subroutine ZERO is used for the special case of zero craft speed. For speeds greater than zero but less than the critical "lift-off" speed, subroutines FOIL, THEO, and EXCIT are used.

```
      SUBROUTINE ZERO(TOOF,TEVF,BOOF,BEVF,GXI,ELL,RHO,NF,Q,TMAS,T,NFREQ,
     2AMP,WAMPL,HOG,NOK)
       COMMON /DFOIL/ GA(6,6),EST(5,30,2),CAL(5,30,2),RAT2,RAT6
      DIMENSION O(10,10),AMP(6),CWPTS(11)
      DIMENSION GB(6,6),GC(6,6),GF(6),T(32,62),GAA(6,6)
      DIMENSION TOOF(6,6),TEVF(6,6),BOOF(6),BEVF(6)
      COMPLEX GB,GC,GF,C,VOA,VOACON
      COMPLEX C1,C2,A3,A4,ARG,HSIN,HCOS,WACCL,WACCM,WA,C3,WVELL,WVELM
      DATA(CWPTS(N),N=1,11)/0.,0.04,0.11,0.31,0.60,0.90,1.13,1.25,1.23
     2,1.15,1.07/
      P1A33=P1A35=P1A22=P1A24=P1A26=P1A44=P2A44=P1A46=P1A55=P1A66=0.
      DO 200 I=1,6
      GF(I)=(0.,0.)
      DO 201 J=1,6
      IF(NFREQ .EQ. 1) GA(I,J)=0.
      GB(I,J)=(0.,0.)
  201 GC(I,J)=(0.,0.)
  200 CONTINUE
C
C     MULTIPLICATION FACTORS FOR NON-DIM. ARE
C         ACCEL FORCES                 (1./MASS)
C         VEL. FORCES                  (1./MASS)*SQRT(LPP/GRAV)
C         DISPL. FORCES                (1./MASS)*(LPP/GRAV)
C       SUBSCRIPTS   11,13,31,33,22
C
C         INERTIA MOMENTS              (1./MASS)/LPP**2
C         ANGULAR VEL. MOMENTS         (1./MASS)*SQRT(LPP/GRAV)/LPP**2
C         ANGULAR DISPL. MOMENTS       (1./MASS)*(LPP/GRAV)/LPP**2
C       SUBSCRIPTS   55,44,46,64,66
C
```

```
C         CROSS INERTIA              (1./MASS)/LPP
C         CROSS VEL.                 (1./MASS)*SQRT(LPP/GRAV)/LPP
C         CROSS DISPL.               (1./MASS)*(LPP/GRAV)/LPP
C       SUBSCRIPTS  15,35,51,53,24,26,42,62
C
C         EXCIT. FORCES/WAVE AMPL. LPP/(MASS*GRAV*WAMPL)
C       SUBSCRIPTS  1,2,3
C
C         EXCIT.MOM./WAVE AMPL.      1./(MASS*GRAV*WAMPL)
C       SUBSCRIPTS  4,5,6
C
      GRAV=32.2
      RMASS=1./TMAS
      ZLDIVG=ELL/GRAV
      ELLSQ=ELL*ELL
      FA1=RMASS
      FA2=FA1/ELLSQ
      FA3=FA1/ELL
      FB1=RMASS*SQRT(ZLDIVG)
      FB2=FB1/ELLSQ
      FB3=FB1/ELL
      FD1=ELL/(TMAS*GRAV*WAMPL)
      FD2=FD1/ELL
C
      C=CMPLX(0.,1.)
      PI=3.14159
      RADDEG=57.29578
      XMU=(181.-HDG)/RADDEG
      SINMU=SIN(XMU)
      COSMU=COS(XMU)
      A1=PI*RHO
      OMEGA=GXI*SQRT(GRAV/ELL)
      OMEG2=OMEGA*OMEGA
      RK=OMEGA*OMEGA/GRAV
      BW1=0.5*A1*OMEGA
C-----------------------------------------------------------------------
C     SUMMATIONS FOR FOIL COEFFICIENTS AND EXCITATION FORCES / MOMENTS
C-----------------------------------------------------------------------
C
      DO 100 I=1,NF
      CPL=Q(1,I)
      SPAN=Q(2,I)
      CHORD=Q(3,I)
      X=Q(4,I)
      Y=Q(5,I)
      Z=Q(6,I)
      DGAMMA=Q(7,I)
      ASP=Q(9,I)
      STRUT=Q(10,I)
      MCPL=CPL
      AREA=SPAN*CHORD
      SPAN3=SPAN**3
      ASPRAT=SPAN/CHORD
      ASPCOR=ASPRAT/(ASPRAT+ASP)
```

```
      CPL=CPL*ASPCOR
      GAMMA=DGAMMA/57.296
      SING=SIN(GAMMA)
      SINGSQ=SING*SING
      COSG=COS(GAMMA)
      COSGSQ=COSG*COSG
C   *    *    *    *    *   (FOIL COEFFICIENTS)   *    *    *    *
C  ADDED MASS
C
      IF(NFREQ .GT. 1) GO TO 308
      A2=0.25*AREA*CHORD*CPL
      P1A33=P1A33+(A2*COSGSQ)
      P1A35=P1A35+(A2*X*COSGSQ)
      P1A55=P1A55+(A2*COSGSQ*(CHORD*CHORD/32.+X*X))
      P1A22=P1A22+(A2*SINGSQ)
      P1A24=P1A24+((A2*Z*SINGSQ)+(A2*Y*SING*COSG))
      P1A26=P1A26+(A2*X*SINGSQ)
      P1A44=P1A44+(A2*((Z*SING+Y*COSG)**2))
      P2A44=P2A44+(AREA*AREA*SPAN/48.)*CPL
      P1A46=P1A46+((A2*Z*X*SINGSQ)+(A2*Y*X*SING*COSG))
      P1A66=P1A66+(A2*SINGSQ*(CHORD*CHORD/32.+X*X))
C
C  DAMPING
C
  308 BWAVE=SW=0.
      IF(STRUT .EQ. 0.) GO TO 38
      SS=(Y+.5*SPAN*COSG)*COSG+(Z+.5*SPAN*SING)*SING
      P=OMEGA*OMEGA*SPAN/GRAV
      L1=10.*P
      IF(L1 .GT. 0) GO TO 30
      CW2=0.
      GO TO 36
   30 IF(L1-10) 32,34,34
   32 P1=L1+1
      P1=P1/10.
      CW2=(10.*(P-P1)*(CWPTS(L1+2)-CWPTS(L1+1)))+CWPTS(L1+2)
      GO TO 36
   34 CW2=1.07/P
   36 BWAVE=BW1*CW2*SPAN*AREA*SINGSQ*CPL
   38 CONTINUE
      ALF=ABS(90.-DGAMMA)
      MCYCL=1
   22 IF(ALF-40.) 10,12,12
   10 CD=.0467*ALF
      GO TO 14
   12 CD=1.17
   14 IF(MCYCL-2) 16,17,18
   16 CDZ=CD
      MCYCL=2
      ALF=DGAMMA
      IF(ALF .GT. 90.) ALF=180.-ALF
      GO TO 22
   17 CDY=CD
      MCYCL=3
```

```
      S=-Y/Z
      ALF=ABS(57.296*ATAN(S)-DGAMMA)
      GO TO 22
   18 CDROLL=CD
C
      COM1=4.*RHO*CHORD*CPL/(3.*PI)
      Q1=Y*COSG+Z*SING
      VOA=C*OMEGA*(-SING*(AMP(2)+X*AMP(6)-Z*AMP(4))+COSG*(AMP(3)+Y*AMP(4
     2)-X*AMP(5)))
      VOACON=-VOA
      VOASQR=SQRT(REAL(VOA*VOACON))
      RELS=SPAN3*REAL(C*OMEGA*AMP(4)*VOACON)/(12.*VOASQR)
      GB(3,3)=GB(3,3)+COM1*CDZ*COSGSQ*VOASQR*SPAN
      GB(5,5)=GB(5,5)+COM1*CDZ*X*X*COSGSQ*VOASQR*SPAN
      IF(STRUT .NE. 0.) SW=BWAVE*SINGSQ
      GB(2,2)=GB(2,2)+COM1*CDY*SINGSQ*VOASQR*SPAN+SW
      IF(STRUT .NE. 0.) SW=BWAVE*ABS(SING*Q1)
      GB(4,2)=GB(4,2)-COM1*CDY*SING*(VOASQR*SPAN*Q1+RELS)
      GB(4,4)=GB(4,4)+COM1*CDROLL*(Q1*(VOASQR*SPAN*Q1+RELS)+RELS*Q1+(SPA
     2N3*VOASQR/12.))+SW
      GB(4,6)=GB(4,6)-COM1*CDY*X*SING*(Q1*VOASQR*SPAN+RELS)
      IF(STRUT .NE. 0.) SW=BWAVE*ABS(X)*SINGSQ
      GB(6,6)=GB(6,6)+COM1*CDY*X*X*SINGSQ*VOASQR*SPAN+SW
C
C    *    *    *    *    *  (EXCITATION FORCES)   *    *    *    *    *
   50 Q2=Y*SINMU+X*COSMU
      C1=-0.25*PI*RHO*(CHORD**2)*WAMPL*OMEG2*CMPLX(COSG,(SINMU*SING))
      C2=CEXP(RK*CMPLX(Z,-Q2))
      A3=C1*C2
      A4=RK*CMPLX(SING,(-SINMU*COSG))
      ARG=.5*SPAN*A4
      HSIN=.5*(CEXP(ARG)-CEXP(-ARG))
      HCOS=.5*(CEXP(ARG)+CEXP(-ARG))
      WACCL=2.*C1*C2*HSIN/A4
      WACCM=(A3/A4)*(SPAN*HCOS-(2.*HSIN/A4))
      WA=WAMPL*OMEGA*CMPLX((-SINMU*SING),COSG)
      ARG=WA*C2
      C3=4.*RHO*CHORD*WA*C2*CSQRT(ARG*CONJG(ARG))/(3.*PI*A4)
      WVELL=C3*(SPAN*REAL(A4)*HCOS+2.*(1.-(REAL(A4)/A4))*HSIN)
      WVELM=C3*((2.*Q1-(2.*(1.+REAL(A4)*Q1)/A4)+.5*SPAN*SPAN*REAL(A4)+(4
     2.*REAL(A4)/(A4*A4)))*HSIN+SPAN*(1.+REAL(A4)*Q1-(2.*REAL(A4)/A4))*H
     3COS)
      GF(2)=GF(2)-(WACCL+WVELL*CDY)*SING*ASPCOR
      GF(3)=GF(3)+(WACCL+WVELL*CDZ)*COSG*ASPCOR
      GF(4)=GF(4)+((WACCL*Q1+WACCM)+WVELM*CDROLL)*ASPCOR
      GF(5)=GF(5)+(WACCL+WVELL*CDZ)*X*COSG*ASPCOR
      GF(6)=GF(6)-(WACCL+WVELL*CDY)*X*SING*ASPCOR
      IF(MCPL .EQ. 1) GO TO 100
```

```
      Y=-Y
      COSG=-COSG
      MCPL=1
      GO TO 50
  100 CONTINUE
C
C-------------------------------------------------------------------
C     FOIL COEFFICIENTS  (NON-DIM.)
C-------------------------------------------------------------------
      IF(NFREQ .GT. 1) GO TO 310
      GA(3,3)=FA1*(+A1*P1A33)
      GA(3,5)=FA3*(-A1*P1A35)
      GA(5,3)=GA(3,5)
      GA(5,5)=FA2*(+A1*P1A55)
      GA(2,2)=FA1*(+A1*P1A22)
      GA(2,4)=FA3*(-A1*P1A24)
      GA(2,6)=FA3*(+A1*P1A26)
      GA(4,2)=GA(2,4)
      GA(4,4)=FA2*(+A1*(P1A44+P2A44))
      GA(4,6)=FA2*(-A1*P1A46)
      GA(6,2)=GA(2,6)
      GA(6,4)=GA(4,6)
      GA(6,6)=FA2*(+A1*P1A66)
  310 GB(3,3)=FB1*GB(3,3)
      GB(5,5)=FB2*GB(5,5)
      GB(2,2)=FB1*GB(2,2)
      GB(4,2)=FB3*GB(4,2)
      GB(4,4)=FB2*GB(4,4)
      GB(4,6)=FB2*GB(4,6)
      GB(6,6)=FB2*GB(6,6)
      DO 400 K=1,6
      T(NFREQ,K)=GA(K,K)
      T(NFREQ,K+10)=REAL(GB(K,K))
      T(NFREQ,K+20)=AIMAG(GB(K,K))
      T(NFREQ,K+30)=REAL(GC(K,K))
  400 T(NFREQ,K+40)=AIMAG(GC(K,K))
      T(NFREQ,7)=GA(3,5)
      T(NFREQ,8)=GA(2,6)
      T(NFREQ,9)=GA(2,4)
      T(NFREQ,10)=GA(4,6)
      T(NFREQ,17)= REAL(GB(3,5))
      T(NFREQ,27)=AIMAG(GB(3,5))
      T(NFREQ,18)= REAL(GB(2,6))
      T(NFREQ,28)=AIMAG(GB(2,6))
      T(NFREQ,19)= REAL(GB(2,4))
      T(NFREQ,29)=AIMAG(GB(2,4))
      T(NFREQ,20)= REAL(GB(4,6))
      T(NFREQ,30)=AIMAG(GB(4,6))
      T(NFREQ,37)= REAL(GC(3,5))
      T(NFREQ,47)=AIMAG(GC(3,5))
      T(NFREQ,38)= REAL(GC(2,6))
      T(NFREQ,48)=AIMAG(GC(2,6))
      T(NFREQ,39)= REAL(GC(2,4))
      T(NFREQ,49)=AIMAG(GC(2,4))
      T(NFREQ,40)= REAL(GC(4,6))
      T(NFREQ,50)=AIMAG(GC(4,6))
```

```
C
C     MULTIPLICATION OF ACCELERATION TERMS BY  -GXI*GXI
C     MULTIPLICATION OF VELOCITY TERMS BY  +GXI
C
      A=-GXI*GXI
      B=GXI
      DO 202 I=1,6
      DO 203 J=1,6
      GAA(I,J)=A*GA(I,J)
  203 GB(I,J)=B*GB(I,J)
  202 CONTINUE
C
C     FOIL COMPONENTS FOR MATRICIES  -TOD AND TEV-
C             TOD(ROW,COL)   ,   TEV(ROW,COL)
C     PRINTOUT OF MATRICIES IS FOR THE FIRST WE(ND) PRINTED
C
      DO 205 I=1,3
      DO 206 J=1,3
      IEV=I+I
      JEV=J+J
      IOD=IEV-1
      JOD=JEV-1
      TODF(I,J)=GAA(IOD,JOD)-AIMAG(GB(IOD,JOD))+REAL(GC(IOD,JOD))
      TODF(I,J+3)=+REAL(GB(IOD,JOD))+AIMAG(GC(IOD,JOD))
      TODF(I+3,J+3)=TODF(I,J)
      TODF(I+3,J)=-TODF(I,J+3)
      TEVF(I,J)=GAA(IEV,JEV)-AIMAG(GB(IEV,JEV))+REAL(GC(IEV,JEV))
      TEVF(I,J+3)=+REAL(GB(IEV,JEV))+AIMAG(GC(IEV,JEV))
      TEVF(I+3,J+3)=TEVF(I,J)
      TEVF(I+3,J)=-TEVF(I,J+3)
  206 CONTINUE
  205 CONTINUE
C-----------------------------------------------------------------
C      EXCITATION FORCES AND MOMENTS   (NON-DIM.)
C-----------------------------------------------------------------
C
      GF(2)=FD1*GF(2)
      GF(3)=FD1*GF(3)
      GF(4)=FD2*GF(4)
      GF(5)=FD2*GF(5)
      GF(6)=FD2*GF(6)
      DO 402 K=1,6
      T(NFREQ,K+50)= REAL(GF(K))
  402 T(NFREQ,K+56)=AIMAG(GF(K))
C
C     FOIL COMPONENTS FOR MATRICIES  -BOD AND BEV-
C
      BODF(1)=0.0
      BODF(2)=+REAL(GF(3))
      BODF(3)=+REAL(GF(5))
      BODF(4)=0.0
      BODF(5)=-AIMAG(GF(3))
      BODF(6)=-AIMAG(GF(5))
      BEVF(1)=+REAL(GF(2))
      BEVF(2)=+REAL(GF(4))
      BEVF(3)=+REAL(GF(6))
      BEVF(4)=-AIMAG(GF(2))
      BEVF(5)=-AIMAG(GF(4))
      BEVF(6)=-AIMAG(GF(6))
      RETURN
  999 END
```

```
      SUBROUTINE FOIL(TODF,TEVF,BODF,BEVF,VKNOTS,WAMPL,HDG1,GXI,ELL,RHO,
     2NF,Q,TMAS,T,NFREQ,WAVL)
      COMMON /QFOIL/ GA(6,6)
      DIMENSION TODF(6,6),TEVF(6,6),BODF(6),BEVF(6),Q(10,10)
      DIMENSION GB(6,6),GC(6,6),GF(6),T(32,62),GAA(6,6)
      COMPLEX CK,B2,C2,P1B33,P1C33,P2B35,P1C35,P2C35,P1B22,P1B24,P1C24,P
     12B26,P1C26,P1B44,P2B44,P1C44,P2B46,P1C46,P1B53,P1C53,P3B55,P1C55,P
     22C55,P1B62,P1B64,P1C64,P2B66,P1C66
      COMPLEX C,EXL,EXM,AA,ARG,HSIN,HCOS,V1,V2,W1,W2,PL1,PF2,PF3,PL2,PF4
     2,PM1,PF5,PF6,BB,W3
      COMPLEX GB,GC,GF
      P1A33=P1A35=P1B35=P1A22=P1A24=P1A26=P1B26=P1A44=P2A44=P1A46=P1B46=
     1P1A55=P1B55=P2B55=P1A66=P1B66=P3B66=0.
      P1B33=P1C33=P2B35=P1C35=P2C35=P1B22=P1B24=P1C24=P2B26=P1C26=P1B44=
     2P2B44=P1C44=P2B46=P1C46=P1B53=P3B55=P1C55=P2C55=P1B62=P1B64=P1C64=
     3P2B66=P1C66=(0.,0.)
      PF2=PF3=PF4=PF5=PF6=(0.,0.)
      DO 200 I=1,6
      GF(I)=(0.,0.)
      DO 201 J=1,6
      IF(NFREQ .EQ. 1) GA(I,J)=0.
      GB(I,J)=(0.,0.)
  201 GC(I,J)=(0.,0.)
  200 CONTINUE
C
C     MULTIPLICATION FACTORS FOR NON-DIM. ARE
C         ACCEL FORCES                (1./MASS)
C         VEL. FORCES                 (1./MASS)*SQRT(LPP/GRAV)
C         DISPL. FORCES               (1./MASS)*(LPP/GRAV)
C       SUBSCRIPTS  11,13,31,33,22
C
C         INERTIA MOMENTS             (1./MASS)/LPP**2
C         ANGULAR VEL. MOMENTS        (1./MASS)*SQRT(LPP/GRAV)/LPP**2
C         ANGULAR DISPL. MOMENTS      (1./MASS)*(LPP/GRAV)/LPP**2
C       SUBSCRIPTS  55,44,46,64,66
C
C         CROSS INERTIA               (1./MASS)/LPP
C         CROSS VEL.                  (1./MASS)*SQRT(LPP/GRAV)/LPP
C         CROSS DISPL.                (1./MASS)*(LPP/GRAV)/LPP
C       SUBSCRIPTS  15,35,51,53,24,26,42,62
C
C         EXCIT. FORCES/WAVE AMPL. LPP/(MASS*GRAV*WAMPL)
C       SUBSCRIPTS  1,2,3
C
C         EXCIT.MOM./WAVE AMPL.     1./(MASS*GRAV*WAMPL)
C       SUBSCRIPTS  4,5,6
C
```

```
      GRAV=32.2
      RMASS=1./TMAS
      ZLDIVG=ELL/GRAV
      ELLSQ=ELL*ELL
      FA1=RMASS
      FB1=RMASS*SQRT(ZLDIVG)
      FC1=RMASS*ZLDIVG
      FA2=FA1/ELLSQ
      FB2=FB1/ELLSQ
      FC2=FC1/ELLSQ
      FA3=FA1/ELL
      FB3=FB1/ELL
      FC3=FC1/ELL
      FD1=ELL/(TMAS*GRAV*HAMPL)
      FD2=FD1/ELL
C
      PI=3.14159
      XMU=(180.0-HDG1)/57.2957795
      U=VKNOTS*1.689
      SINMU=SIN(XMU)
      COSMU=COS(XMU)
      OMEGA=SQRT(2.*PI*GRAV/WAVL)
      OMEGAE=SXI*SQRT(GRAV/ELL)
      A1=PI*RHO
      A3=A1*U
      B1=0.5*RHO*U
      C1=B1*U
      BB=CMPLX(0.,1.)
C-----------------------------------------------------------------------
C     SUMMATIONS FOR FOIL COEFFICIENTS AND EXCITATION FORCES / MOMENTS
C-----------------------------------------------------------------------
C
      DO 100 I=1,NF
      CPL=Q(1,I)
      SPAN=Q(2,I)
      CHORD=Q(3,I)
      S=Q(4,I)
      Y=Q(5,I)
      Z=Q(6,I)
      DGAMMA=Q(7,I)
      CLZ=Q(8,I)
      ASP=Q(9,I)
      NCPL=CPL
      AREA=SPAN*CHORD
      ASPRAT=SPAN/CHORD
      ASPCOR=ASPRAT/(ASPRAT+ASP)
      CPL=CPL*ASPCOR
      GAMMA=DGAMMA/57.2957795
      SING=SIN(GAMMA)
      SINGSQ=SING*SING
      COSG=COS(GAMMA)
      COSGSQ=COSG*COSG
      SINMUG=SINMU*SING
      CLALPH=2.*PI
```

```
      XK1=0.5*OMEGAE*CHORD/U
      XK2=(OMEGA*OMEGA)/GRAV
      XK3=0.5*(OMEGAE*OMEGAE)*CHORD*COSMU/GRAV
      CALL THEO(XK1,CK)
      A2=0.25*AREA*CHORD*CPL
      B2=AREA*CLALPH*CK*CPL
      C2=CLZ*AREA*CK*CPL
C    *    *    *    *    *    *    *    *    *    *    *    *    *
      IF(NFREQ .GT. 1) GO TO 308
      P1A33=P1A33+(A2*COSGSQ)
      P1A35=P1A35+(A2*S*COSGSQ)
      P1A55=P1A55+(A2*COSGSQ*(CHORD*CHORD/32.+S*S))
      P1A22=P1A22+(A2*SINGSQ)
      P1A24=P1A24+((A2*Z*SINGSQ)+(A2*Y*SING*COSG))
      P1A26=P1A26+(A2*S*SINGSQ)
      P1A44=P1A44+(AREA*AREA*SPAN/48.)*CPL
      P2A44=P2A44+(A2*((Z*SING+Y*COSG)**2))
      P1A46=P1A46+((A2*Z*S*SINGSQ)+(A2*Y*S*SING*COSG))
      P1A66=P1A66+(A2*SINGSQ*(CHORD*CHORD/32.+S*S))
  308 P1B33=P1B33+(B2*COSGSQ)
      P1B35=P1B35+(A2*COSGSQ)
      P2B35=P2B35+(B2*COSGSQ*(S+(CHORD/4.)))
      P1B53=P1B53+(B2*(S-(CHORD/4.))*COSGSQ)
      P1B55=P1B55+(A2*S*COSGSQ)
      P2B55=P2B55+((CHORD**3)*SPAN*COSGSQ/16.)*CPL
      P3B55=P3B55+(B2*(S+(CHORD/4.))*(S-(CHORD/4.))*COSGSQ)
      P1B22=P1B22+(B2*SINGSQ)
      P1B24=P1B24+((B2*Z*SINGSQ)+(B2*Y*SING*COSG))
      P1B26=P1B26+(A2*SINGSQ)
      P2B26=P2B26+(B2*(S+(CHORD/4.))*SINGSQ)
      P1B44=P1B44+(B2*SPAN*SPAN/12.)
      P2B44=P2B44+(B2*((Z*SING+Y*COSG)**2))
      P1B46=P1B46+((A2*Z*SINGSQ)+(A2*Y*SING*COSG))
      P2B46=P2B46+(B2*(S+(CHORD/4.))*((Z*SINGSQ)+(Y*SING*COSG)))
      P1B62=P1B62+(B2*(S-(CHORD/4.))*SINGSQ)
      P1B64=P1B64+(B2*(S-(CHORD/4.))*((Z*SINGSQ)+(Y*SING*COSG)))
      P1B66=P1B66+(A2*S*SINGSQ)
      P2B66=P2B66+(B2*(S+(CHORD/4.))*(S-(CHORD/4.))*SINGSQ)
      P3B66=P3B66+((CHORD**3)*SPAN*SINGSQ/16.)*CPL
      P1C33=P1C33+(C2*COSG)
      P1C35=P1C35+(B2*COSGSQ)
      P2C35=P2C35+(C2*(S-(CHORD/4.))*COSG)
      P1C53=P2C35
      P1C55=P1C55+(B2*(S-(CHORD/4.))*COSGSQ)
      P2C55=P2C55+(C2*(S+(CHORD/4.))*(S-(CHORD/4.))*COSG)
      P1C24=P1C24+(C2*Y*SING)
      P1C26=P1C26+(B2*SINGSQ)
      P1C64=P1C64+(C2*(S-(CHORD/4.))*Y*SING)
      P1C66=P1C66+(B2*(S-(CHORD/4.))*SINGSQ)
      P1C44=P1C44+(C2*Y*((Y*COSG)+(Z*SING)))
      P1C46=P1C46+((B2*Z*SINGSQ)+(B2*Y*SING*COSG))
```

```
C   *    *    *    *    *    *    *    *    *    *    *    *    *
      CALL EXCIT(XK3,XK1,CK,EXL,EXM)
   25 C=CMPLX(COSG,SINMUG)
      XREAL=XK2*SING
      XIMAG=XK2*SINMU*COSG
      AA=CMPLX(XREAL,-XIMAG)
      ARG=0.5*AA*SPAN
      HSIN=0.5*(CEXP(ARG)-CEXP(-ARG))
      HCOS=0.5*(CEXP(ARG)+CEXP(-ARG))
      V1=(2./AA)*HSIN
      V2=(1./(AA*AA))*(AA*SPAN*HCOS-2.*HSIN)
      XREAL=XK2*Z
      XIMAG=-XK2*Y*SINMU
      AA=CMPLX(XREAL,XIMAG)
      AA=WAMPL*OMEGA*CEXP(AA)*C
      W1=AA*V1*BB
      W2=AA*V2*BB
      W3=AA*BB
      IF(NCPL-1) 51,51,50
   50 W1=2.*W1
      W2=2.*W2
      W3=2.*W3
   51 XIMAG=-XK2*S*COSMU
      AA=CEXP(CMPLX(0.,XIMAG))
      C=CHORD*EXL*AA
      PL1=C*AIMAG(W1)*BB
      PF3=PF3+(PL1*COSG*ASPCOR)
      PL1=C*REAL(W1)
      PL2=CHORD*W2*EXL
      PM1=0.25*CHORD*CHORD*EXM*AA
      PF5=PF5-(((-S*PL1)-PM1*BB*AIMAG(W3))*COSG)*ASPCOR
      HD=HDG1
      IF(HD .GT. 180.) HD=HD-180.
      IF(HD .GT. 172.) GO TO 100
      IF(HD .LT. 8.) GO TO 100
      PF2=PF2-(PL1*SING*ASPCOR)
      PF4=PF4+(PL2+PL1*(Y*COSG+Z*SING))*ASPCOR
      PF6=PF6+(((S*PL1)+PM1*REAL(W3))*SING)*ASPCOR
  100 CONTINUE
C
C-----------------------------------------------------------------------
C     FOIL COEFFICIENTS  (NON-DIM.)
C-----------------------------------------------------------------------
      IF(NFREQ .GT. 1) GO TO 310
      GA(3,3)=FA1*(+A1*P1A33)
      GA(3,5)=FA3*(-A1*P1A35)
      GA(5,3)=GA(3,5)
      GA(5,5)=FA2*(+A1*P1A55)
      GA(2,2)=FA1*(+A1*P1A22)
      GA(2,4)=FA3*(-A1*P1A24)
      GA(2,6)=FA3*(+A1*P1A26)
      GA(4,2)=GA(2,4)
      GA(4,4)=FA2*(+A1*(P1A44+P2A44))
      GA(4,6)=FA2*(-A1*P1A46)
```

```
      GA(6,2)=GA(2,6)
      GA(6,4)=GA(4,6)
      GA(6,6)=FA2*(+A1*P1A66)
  310 GB(3,3)=FB1*(+B1*P1B33)
      GB(3,5)=FB3*(-A3*P1B35-B1*P2B35)
      GB(5,3)=FB3*(-B1*P1B53)
      GB(5,5)=FB2*(+A3*P1B55+A3*P2B55+B1*P3B55)
      GB(2,2)=FB1*(+B1*P1B22)
      GB(2,4)=FB3*(-B1*P1B24)
      GB(2,6)=FB3*(+A3*P1B26+B1*P2B26)
      GB(4,2)=GB(2,4)
      GB(4,4)=FB2*(+B1*(P1B44+P2B44))
      GB(4,6)=FB2*(-A3*P1B46-B1*P2B46)
      GB(6,2)=FB3*(+B1*P1B62)
      GB(6,4)=FB2*(-B1*P1B64)
      GB(6,6)=FB2*(+A3*P1B66+B1*P2B66+A3*P3B66)
      GC(3,3)=FC1*(+C1*P1C33)
      GC(3,5)=FC3*(-C1*(P1C35+P2C35))
      GC(5,3)=FC3*(-C1*P1C53)
      GC(5,5)=FC2*(+C1*(P1C55+P2C55))
      GC(2,4)=FC3*(-C1*P1C24)
      GC(2,6)=FC3*(+C1*P1C26)
      GC(4,4)=FC2*(+C1*P1C44)
      GC(4,6)=FC2*(-C1*P1C46)
      GC(6,4)=FC2*(-C1*P1C64)
      GC(6,6)=FC2*(+C1*P1C66)
      DO 400 K=1,6
      T(NFREQ,K)=GA(K,K)
      T(NFREQ,K+10)=REAL(GB(K,K))
      T(NFREQ,K+20)=AIMAG(GB(K,K))
      T(NFREQ,K+30)=REAL(GC(K,K))
  400 T(NFREQ,K+40)=AIMAG(GC(K,K))
      T(NFREQ,7)=GA(3,5)
      T(NFREQ,8)=GA(2,6)
      T(NFREQ,9)=GA(2,4)
      T(NFREQ,10)=GA(4,6)
      T(NFREQ,17)= REAL(GB(3,5))
      T(NFREQ,27)=AIMAG(GB(3,5))
      T(NFREQ,18)= REAL(GB(2,6))
      T(NFREQ,28)=AIMAG(GB(2,6))
      T(NFREQ,19)= REAL(GB(2,4))
      T(NFREQ,29)=AIMAG(GB(2,4))
      T(NFREQ,20)= REAL(GB(4,6))
      T(NFREQ,30)=AIMAG(GB(4,6))
      T(NFREQ,37)= REAL(GC(3,5))
      T(NFREQ,47)=AIMAG(GC(3,5))
      T(NFREQ,38)= REAL(GC(2,6))
      T(NFREQ,48)=AIMAG(GC(2,6))
      T(NFREQ,39)= REAL(GC(2,4))
      T(NFREQ,49)=AIMAG(GC(2,4))
      T(NFREQ,40)= REAL(GC(4,6))
      T(NFREQ,50)=AIMAG(GC(4,6))
```

```
C
C     MULTIPLICATION OF ACCELERATION TERMS BY  -GXI*GXI
C     MULTIPLICATION OF VELOCITY TERMS BY  +GXI
C
      A=-GXI*GXI
      B=GXI
      DO 202 I=1,6
      DO 203 J=1,6
      GAA(I,J)=A*GA(I,J)
  203 GB(I,J)=B*GB(I,J)
  202 CONTINUE
C
C     FOIL COMPONENTS FOR MATRICIES  -TOD AND TEV-
C
      DO 205 I=1,3
      DO 206 J=1,3
      IEV=I+I
      JEV=J+J
      IOD=IEV-1
      JOD=JEV-1
      TODF(I,J)=GAA(IOD,JOD)-AIMAG(GB(IOD,JOD))+REAL(GC(IOD,JOD))
      TODF(I,J+3)=+REAL(GB(IOD,JOD))+AIMAG(GC(IOD,JOD))
      TODF(I+3,J+3)=TODF(I,J)
      TODF(I+3,J)=-TODF(I,J+3)
      TEVF(I,J)=GAA(IEV,JEV)-AIMAG(GB(IEV,JEV))+REAL(GC(IEV,JEV))
      TEVF(I,J+3)=+REAL(GB(IEV,JEV))+AIMAG(GC(IEV,JEV))
      TEVF(I+3,J+3)=TEVF(I,J)
      TEVF(I+3,J)=-TEVF(I,J+3)
  206 CONTINUE
  205 CONTINUE
C-----------------------------------------------------------------
C      EXCITATION FORCES AND MOMENTS  (NON-DIM.)
C-----------------------------------------------------------------
C
      GF(2)=FD1*(+A3*PF2)
      GF(3)=FD1*(+A3*PF3)
      GF(4)=FD2*(+A3*PF4)
      GF(5)=FD2*(+A3*PF5)
      GF(6)=FD2*(+A3*PF6)
      DO 402 K=1,6
      T(NFREQ,K+50)= REAL(GF(K))
  402 T(NFREQ,K+56)=AIMAG(GF(K))
C
C     FOIL COMPONENTS FOR MATRICIES  -BOD AND BEV-
C
      BODF(1)=0.0
      BODF(2)=+REAL(GF(3))
      BODF(3)=+REAL(GF(5))
      BODF(4)=0.0
      BODF(5)=-AIMAG(GF(3))
      BODF(6)=-AIMAG(GF(5))
      BEVF(1)=+REAL(GF(2))
      BEVF(2)=+REAL(GF(4))
      BEVF(3)=+REAL(GF(6))
      BEVF(4)=-AIMAG(GF(2))
      BEVF(5)=-AIMAG(GF(4))
      BEVF(6)=-AIMAG(GF(6))
      RETURN
  999 END
```

```
      SUBROUTINE THEO(XK1,CK)
      COMPLEX CK
 1001 FORMAT(*      J0    IER=*,I2)
 1002 FORMAT(*      J1    IER=*,I2)
 1003 FORMAT(*      Y0    IER=*,I2)
 1004 FORMAT(*      Y1    IER=*,I2)
      CALL IBESJ(XK1,0,XJ0,1.E-6,IER)
      IF(IER.GE.3) GO TO 77
      CALL IBESJ(XK1,1,XJ1,1.E-6,IER)
      IF(IER.GE.3) GO TO 78
      CALL IBESY(XK1,0,Y0,IER)
      IF(IER.EQ.3) GO TO 79
      CALL IBESY(XK1,1,Y1,IER)
      IF(IER.EQ.3) GO TO 80
      T1=XJ1+Y0
      T2=Y1-XJ0
      X=XJ1*T1+Y1*T2
      Y=-Y1*Y0-XJ1*XJ0
      CK=CMPLX(X,Y)
      X=T1*T1+T2*T2
      CK=CK/X
      GO TO 81
   77 WRITE(6,1001) IER
      GO TO 81
   78 WRITE(6,1002) IER
      GO TO 81
   79 WRITE(6,1003) IER
      GO TO 81
   80 WRITE(6,1004) IER
   81 RETURN
      END
```

```
      SUBROUTINE EXCIT(XK3,XK1,CK,EXL,EXM)
      COMPLEX CK,T1,T2,T3,T4,EXL,EXM,V
 1001 FORMAT(*      J0    IER=*,I2)
 1002 FORMAT(*      J1    IER=*,I2)
 1003 FORMAT(*      J2    IER=*,I2)
 1004 FORMAT(*      J3    IER=*,I2)
      IF(XK3) 2,3,3
    2 ISIGN=0
      GO TO 4
    3 ISIGN=1
    4 XK3=ABS(XK3)
      CALL IBESJ(XK3,0,XJ0,1.E-4,IER)
      IF(IER.GE.3) GO TO 77
      CALL IBESJ(XK3,1,XJ1,1.E-6,IER)
      IF(IER.GE.3) GO TO 78
      CALL IBESJ(XK3,2,XJ2,1.E-6,IER)
      IF(IER.GE.3) GO TO 79
      CALL IBESJ(XK3,3,XJ3,1.E-6,IER)
      IF(IER.GE.3) GO TO 80
      IF(ISIGN) 5,5,6
    5 XJ1=-XJ1
      XJ3=-XJ3
      XK3=-XK3
    6 T1=CMPLX(XJ0,-XJ1)
      T2=T1*CK
      R1=0.5*XK1*(XJ0+XJ2)
      T3=CMPLX(0.,R1)
      EXL=T2+T3
      T1=XJ0*CK
      T2=XJ1*(1.-CK)
      V=CMPLX(0.,1.)
      T2=V*T2
      R1=(XJ1+XJ3)*(XK1/4.)
      T3=CMPLX(R1,0.)
      T4=CMPLX(XJ2,0.)
      EXM=T1+T2-T3+T4
      GO TO 81
   77 WRITE(6,1001) IER
      GO TO 81
   78 WRITE(6,1002) IER
      GO TO 81
   79 WRITE(6,1003) IER
      GO TO 81
   80 WRITE(6,1004) IER
   81 RETURN
      END
```

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